765

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DISCUSSION OF PROCEEDINGS PAPERS

132, 313, 449, 468, 540, 583, 586, 588,

STRUCTURAL DIVISION

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Current discussion of papers sponsored by the Structural Division is presented as follows:

Number		Page
132	Specifications for Structures of a Moderate Strength Aluminum Alloy of High Resistance to Corrosion: Progress Report of the Committee of the Structural Division on Design in Lightweight Structural Alloys. (May, 1952. Prior discussion: None. Discussion closed)	
	Corrections	1
313	General Design of the East Delaware Water Supply Tunnel, by Stanley M. Dore. (October, 1953. Prior discussion: 488. Discussion closed. There will be no closure)	
449	Non-Uniform Torsion of Plate Girders, by Gerald G. Kubo, Bruce G. Johnston, and William J. Eney. (June, 1954. Prior discussion: 563. Discussion closed)	
	Kubo, Gerald G., Johnston, Bruce G., and Eney, William J. (Closure)	9
	Corrections	10
468	Safety and the Probability of Structural Failure, A. M. Freudenthal. (August, 1954. Prior discussion: 654, 664. Discussion closed)	
	Freudenthal, A. M. (Closure)	11
540	The Vibration of Steel Stacks, by Walter L. Dickey and Glenn B. Woodruff. (November, 1954. Prior discussion: 661. Discussion closed)	
	Dickey, Walter L., and Woodruff, Glenn B. (Closure)	15
	Corrections	16
583	Ultimate Slopes and Deflections—A Brief for Limit Design, by George C. Ernst. (January, 1955. Prior discussion: None. Discussion closed)	
	Diwan, A.	17
586	Multiple Span Gabled Frames, by John D. Griffiths. (January, 1955. Prior discussion: None. Discussion closed)	
	Diwan, A. F.	31
588	Continuous Prestressing, by Robert B. B. Moorman. (January, 1955. Prior discussion: None. Discussion closed)	
	Diwan, A.	45
	Mensch, L. J.	49

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Discussion of

SPECIFICATIONS FOR STRUCTURES OF A MODERATE STRENGTH ALUMINUM ALLOY OF HIGH RESISTANCE TO CORROSION: PROGRESS REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON DESIGN IN LIGHTWEIGHT STRUCTURAL ALLOYS*

(Proc. Paper 132)

INTRODUCTION

CORRECTIONS.—A progress report of this committee entitled "Specifications for Structures of a Moderate Strength Aluminum Alloy of High Resistance to Corrosion," was published as Separate No. 132, Proceedings, ASCE, May, 1952. These specifications cover allowable stresses, design rules and fabrication procedures for structures built of the aluminum alloy known commercially as 6061-T6 (formerly 61S-T6). The basic allowable tensile working stress is 15 kips per sq in. based on a minimum yield strength of 35 kips per sq in. and a minimum tensile strength of 38 kips per sq in.

Recently this Committee has reviewed Separate No. 132 to determine what modifications are desirable in order to bring the specifications up to date, to make them more useful, or to improve their clarity. The revisions that the Committee has recommended as a result of its study are listed in this report.

The most significant changes concern the following items:

1) Maximum Slenderness Ratio. The original specifications required that the slenderness ratio for compression members should not exceed 120. The new specifications permit the slenderness ratio to exceed 120 provided that special care is taken to insure that the member has adequate strength to resist bending loads such as wind and dead load as well as axial forces. The purpose of this revision is to allow the designer a greater freedom of choice in selecting sizes of members, while still retaining an adequate factor of safety. The provision for limiting the slenderness ratio of tension members has been altered in the same way.

2) Combined Compression and Bending. The two formulas that governed the design of beam-columns in Separate No. 132 have been replaced by a single formula that applies to all types of beam-columns. This change is based on the results of recent research on members subjected to combined bending and compression. It considerably simplifies the task of designing such members.

3) Painting. This section has been expanded to include some items not covered in Separate No. 132. The additions include a paragraph on the preparation of aluminum surfaces to be imbedded in concrete and a reference to a specification covering priming paint for steel surfaces to be placed in contact with aluminum.

The references to other paint specifications have been brought up to date.

4) Allowable Stresses for Welded Parts. Separate No. 132 contained a chart which gave allowable stresses in the vicinity of welds. These allowable stresses varied linearly between the relatively low stress permitted at the weld to the higher stress allowed for the parent material at a sufficient distance from the weld that the strength is not reduced by the heat of welding.

The "heat-affected zone" was assumed to extend a distance of 16t on either side of the weld, where t is the thickness of the material.

The revised specifications provide that the reduced allowable stresses that govern at the weld shall apply throughout the entire heat-affected zone, and the extent of the latter is taken to be (3+3t) on either side of the weld. These changes are in accordance with experience gained in welding this material since the preparation of Separate No. 132. The chart of allowable stresses in welded parts has been eliminated in the revised specifications.

New references on welding have been added to the explanation of this section.

5) Welding Procedure. The revised specification permits the use of either aluminum alloy 4043 (formerly 43S) or 5154 (formerly A54S) as filler metal in welding. The latter filler metal was not included in Separate No. 132 because it had not been widely used at the time the original specifications were prepared. Introduction of alloy 5154 as a filler metal makes possible welds of greater strength and ductility than are generally obtained with alloy 4043.

6) Qualification of Welding Procedure and Welding Operator. The minimum requirement for tensile strength to be achieved in a reduced-section tensile test has been increased from 22 to 24 kips per sq in. The latter is the same as the minimum requirement in the ASME Boiler and Pressure Vessel Code.

7) New Alloy Designation. The specifications are affected by a change in alloy designation made by the Aluminum Association, effective October 1, 1954. The new and old designations for the various alloys and tempers mentioned in Separate No. 132 are listed in the following table:

Former Alloy Designation	New Alloy Designation
61S-T6	6061-T6
61S-T4	6061-T4
61S-T43	6061-T43
24S-T4	2024-T4
43S	4043

The preceding changes are to be made wherever alloy designations appear in Separate No. 132.

Since the new alloy designation is uniform throughout the industry, it is advantageous to include the alloy number in the title of the Specifications. Hence, the title has been changed to "Specifications for Structures of Aluminum Alloy 6061-T6 (A Moderate Strength Alloy of High Resistance to Corrosion)."

The revisions in the specifications (except for the changes in alloy designation) are described in more detail in the next section of this report. For the sake of brevity, no attempt has been made to compare the new and old versions. For the original wording of the various sections mentioned in the list of revisions, the reader is referred to Separate No. 132. It is anticipated that the specifications will eventually be published complete in their revised form.

LIST OF REVISIONS

Page 1. Change Reference (1) to "Specifications for Heavy Duty Structures of High-Strength Aluminum Alloy," final report of the Committee of the Structural Division on Design in Lightweight Structural Alloys, Transactions, ASCE, Vol. 117, 1952, p. 1253.

Page 2. After the fourth line insert the following paragraph:

"It is believed that the designer can make more efficient use of a set of specifications if he knows the basis for its various provisions. For this reason, Part IV, Explanation of Specifications, has been added. Part IV contains background information and references concerning those paragraphs of the specifications for which some explanation seems required."

Page 4. In the fifth line, replace the words, "a specified" with the words, "an expected."

Page 5.

Replace Article B-2 with the following:

B-2. Columns with Slenderness Ratio Exceeding 120. Because long columns are relatively flexible, they may be appreciably weakened by the presence of lateral loads that would have little effect on the column strength of stiffer members. For this reason, columns with slenderness ratios greater than 120 should not be used unless special care is taken to insure that the effect of any lateral loads to which the member may be subjected, such as wind, dead load, or the weight of workmen and equipment, are taken into account by using the provision for combined compression and bending in Specification B-7.

Page 6. In the fourth line of Specification B-5, delete the words, "In main members," so that the sentence begins, "The length of the end stay plates. . ."

Replace Article B-7 with the following:

B-7. Combined Compression and Bending.—The allowable stress in a member that carries bending moment in addition to uniform compression (as, for example, an eccentrically loaded column) shall be determined from the following formula:

$$f_b = f_B \left(1 - \frac{P/A}{f_C}\right) \quad \left(1 - \frac{P/A}{f_{CE}}\right)$$

in which (in kips per sq in.):

f_b is the maximum bending stress (compression) that may be permitted at or near the center of the unsupported length, in addition to uniform compression, P/A;

P/A is the average compressive stress on the gross cross section, A, of the member, produced by a column load, P;

f_B is the allowable compressive working stress for the member considered as a beam;

f_C is the allowable working stress for the member considered as an axially loaded column; and

$$f_{CE} = \frac{70{,}000}{(L/r)^2}$$

in which L/r is the slenderness ratio for the member considered as a column tending to fail in the plane of the bending forces.

Page 7. In the second line after formula (3b) in Article B-8, omit the word "transverse" between the words "maximum" and "shear."

Page 8. Replace the definition of L at the top of the page with the following:

L is the laterally unsupported length of beam (clear distance between supports at which the beam is prevented from lateral displacement), or, in the case of a cantilever beam with one end free, L is four thirds of the laterally unsupported length, in inches.

Page 11. In the last sentence of Article D-2d, change the clause that begins with "including" and continues to the end of the paragraph to: "including in each flange area not only the flange proper but also one-sixth of the area of the web."

Page 12. Replace paragraph D-2f with the following:

f.—Steps, c, d and e provide a suitable factor of safety against collapse of the member as a whole, but do not necessarily provide complete protection against local buckling of individual flat surfaces at the design load. Where local buckling at the design load cannot be tolerated because of appearance, or for other reasons, the computed compressive stress on the gross area, f_c, shall not exceed $1.5f_1$, where f_1 is the compressive stress given in Fig. 5 or Fig. 6 for the b/t ratio in question. Regardless of whether or not this limitation is placed on f_c, the member must still be checked by the method outlined in Steps a through e to insure an adequate factor of safety against collapse.

Change the last sentence of Article F-1 to: "In determining this ratio the tension flange shall be considered to consist of the flange angles and cover plates plus one-sixth of the web."

Page 20. Replace Article H-2 with the following:

H-2. Slenderness Ratio of Tension Members.—Long slender members have little resistance to lateral loads. Therefore, tension members with values of slenderness ratio L/r greater than 150 shall not be used unless special care is taken to insure that such members are designed to resist any lateral loads such as wind, dead load, or the weight of workmen and equipment. Stresses caused by the combined bending and tensile loadings shall not exceed allowable limits.

The above change eliminates Eq. 8, so all subsequent equation numbers should be reduced by one.

Page 24. Replace Article I-6 with the following:

I-6. Painting.—Structures of the alloy covered by these Specifications are not ordinarily painted except where (a) the aluminum alloy parts are in contact with, or are fastened to, steel members or other dissimilar materials, or (b) the structures are to be exposed to extremely corrosive conditions. Painting procedure is covered in the following paragraphs, and methods of cleaning and preparation are found in Specification I-7. (Treatment and painting of the structure in accordance with United States Military Specification MIL-T-704 is also acceptable).

a.—Where the aluminum alloy parts are in contact with, or are fastened to, steel members or other dissimilar materials, the aluminum shall be kept from direct contact with the steel or other dissimilar material by painting as described below:

1) Aluminum surfaces to be placed in contact with steel shall be given one coat of zinc chromate primer in accordance with United States Joint Army-Navy Specification JAN-P-735 or the equivalent. Zinc chromate paint shall be allowed to dry before assembly of the parts. Additional protection may be provided by the application of one coat of a suitable nonhardening joint compound, in addition to the zinc chromate primer. The steel surfaces to be placed in contact with aluminum shall be painted with good quality priming paint, such as red lead conforming to Federal Specification TT-P-86a, followed by one coat of paint consisting of 2 lb of aluminum paste pigment (ASTM Specification D962-49, Type II, Class B) per gallon of varnish meeting Federal Specification TT-V-81b, Type II, or the equivalent.

2) Aluminum surfaces to be placed in contact with wood, concrete, or masonry construction, except where the aluminum is to be imbedded in concrete, shall be given a heavy coat of an alkali-resistant bituminous paint before installation. The bituminous paint used shall meet the requirements of United States Military Specification MIL-P-6883. The paint shall be applied as it is received from the manufacturer without the addition of any thinner.

3) Aluminum surfaces to be imbedded in concrete shall, before installation, be given one coat of zinc chromate primer in accordance with United States Joint Army-Navy Specification JAN-P-735 or the equivalent. The paint shall be allowed to dry before the aluminum is placed in the concrete.

b.—Although structures of the alloy covered by these specifications are not ordinarily painted, there may be applications where the structures are to be exposed to extremely corrosive conditions that make over-all painting advisable. In such instances all metal surfaces including contacting surfaces shall, before assembly, be given one shop coat of zinc chromate primer in accordance with United States Joint Army-Navy Specification JAN-P-735 or the equivalent. Zinc chromate paint shall be allowed to dry before assembly of the parts. After assembly, the parts shall be given a second shop coat of paint consisting of 2 lb of aluminum paste pigment (ASTM Specification D962-49, Type II, Class B) per gallon of varnish meeting Federal Specification TT-V-81b, Type II, or the equivalent. Sufficient Prussian blue shall be added to permit detection of an incomplete application of the subsequent paint coat. After erection, bare spots shall be touched up with zinc chromate primer followed by a touch-up coat of aluminum paint as specified above. The completed structure shall be finished according to one of the following methods:

1) One field coat of aluminum paint as specified above, except that Prussian blue shall be omitted from the field coat.

2) One or more field coats of alkyd base enamel pigmented to meet a desired color scheme.

Page 25. Replace Section J with the following:

Section J. Allowable Stresses for Welded Parts

Although the strength of alloy 6061-T6 is lowered locally by the heat of welding, its resistance to corrosion is not impaired and excellent results can be obtained. Welded members of alloy 6061-T6 are proportioned in accordance with the general rules of Part II, except that it is necessary to lower the allowable stresses in the heat-affected zone.

The heat-affected zone shall be considered to extend for a distance, in inches, equal to (3+3t) on either side of the center line of the weld, where t is the thickness in inches of the material adjacent to the weld.

The allowable stresses in the zone extending for a distance of (3+3t) on either side of the weld are as follows:

J-1. The basic allowable stress in tension is 8 kips per sq in.

J-2. The basic allowable stress in bearing against rivets, milled stiffeners, turned bolts in reamed holes and other parts in fixed contact is 15 kips per sq in.

J-3. Allowable compressive stresses are found from Figs. 2, 3, 4 and 8 in the usual manner except that a horizontal cutoff line is to be drawn at a stress of 8 kips per sq in., and all values above this cutoff line are to be ignored. Figs. 5 and 6, which are used for the determination of the stress f_1 in Formula 4, also have a horizontal cutoff line at 8 kips per sq in., and all values above this line are to be ignored.

J-4. Allowable shear stresses are taken from Fig. 7 in the usual manner except that a horizontal cutoff line is to be drawn at a stress of 5 kips per sq in., and all values above this cutoff line are to be ignored.

J-5. Fillet welds shall be designed using an allowable shear stress on the throat of the weld of 4 kips per sq in.

Page 26. Delete Fig. 11

Page 27. In the third line of Article L-3 after "4043," insert "(ASTM designation S5B), 5154 (ASTM designation GR40A)."

In Article L-4, under minimum requirements for test results, change the figure following the words "Reduced Section Tensile Test" from "22" to "24."

Page 29. Replace the explanation of Article B-7 with the following:

B-7. Formula for Combined Compression and Bending. Eq. 1 is based on an interaction formula that gives good agreement with the results of tests on aluminum alloy members subjected to combined compressive end load and bending.[6,7] The formula is applicable to members in which the bending is applied about either the strong or the weak axis.

In the last line of the explanation of Article B-8 change the superscript after the word "elsewhere" from 6 to 8."

Change references 6, 7 and 8 to the following:

6. "Plastic Buckling of Eccentrically Loaded Aluminum Alloy Columns," by J. W. Clark, Proceedings, ASCE, Separate No. 299, Vol. 79, October, 1953.

7. "Designing Aluminum Alloy Members for Combined End Load and Bending," by H. N. Hill, E. C. Hartmann, and J. W. Clark, Proceedings, ASCE, Separate No. 300, Vol. 79, October, 1953.

8. Discussion by E. C. Hartmann of "Rational Design of Steel Columns," by D. H. Young, Transactions, ASCE, Vol. 101, 1936, pp. 475-481.

Replace the first paragraph of the explanation of Article C-1 with the following:

C-1. The curve in Fig. 3 is based on the theoretical solution for the critical bending moment in I-beams as given by S. Timoshenko.[9] It represents a factor of safety of 2.5 applied to the buckling strength of beams subjected to a uniform bending moment. It is assumed that at the ends of the laterally unsupported length there is partial restraint against rotation about a vertical axis and complete restraint against lateral displacement and against rotation about a horizontal axis parallel to the web. The effect of partial restraint at the ends of the span has been taken into account by substituting 0.75L for the length in the formula for buckling of the beam under uniform bending moment.

The part of the curve for values of $\frac{L}{\sqrt{B/S_c}}$ greater than 35 is based on elastic action, whereas the remainder is simply an extension of the same formula using tangent modulus rather than initial modulus. The curve has a cutoff at the basic allowable compressive design stress of 14 kips per sq in. It is important to note that the term L is defined as "laterally unsupported length," which is not necessarily the same as the span of the beam or girder.

The case of uniform bending moment on a simple beam has been used in setting up Fig. 3, because it is a good approximation of conditions frequently encountered in actual design, and because it is somewhat more conservative than many of the other cases that might have been selected. The provision that four-thirds times the length shall be used for L in the case of a cantilever beam has the effect of canceling the length reduction coefficient of 0.75 that was introduced to take account of the effect of lateral restraint in simple beams. Thus, the buckling strength of a cantilever beam is assumed to be equal to the strength of a simply supported beam of the same length subjected to uniform bending moment. This gives a conservative approximation to the strength of a cantilever beam.[9]

Page 31. After the second line, insert the following paragraph:

The limitation placed on the value of f_c in step f of Specification D-2 is intended to provide a factor of safety of at least 1.67 against local buckling at the design load.

Pages 32 and 33. Delete the explanation of Article H-2.

Page 33. Replace the explanation of Section J with the following:

Section J. Allowable Stresses for Welded Parts

The heat of welding lowers the strength of 6061-T6 parts in the vicinity of the weld by an amount that depends on how much heat is applied to the metal in making the weld. On the basis of tests of 6061-T6 welded joints,[29] and the fact that Article L-4 specifies 24 kips per sq in. as the minimum tensile strength requirement to be met in a welding qualification test, the minimum tensile strength in the heat-affected zone has been assumed to be 24 kips per sq in. for design purposes. The corresponding minimum yield strength and shear strength for design purposes have been selected as 15 kips per sq in. and 13 kips per sq in., respectively.

Tests have shown that the extent of the region in which the strength of 6061-T6 parts is reduced by the heat of welding does not exceed a distance of (3+3t) on either side of the center line of the weld. Therefore, design stresses for material at greater distances from the weld are based on the strength of the fully heat-treated metal.

29. "The Strength and Ductility of Welds in Aluminum Alloy Plate" by F. G. Nelson, Jr., and F. M. Howell, The Welding Journal Research Supplement, September, 1952, pp. 3-7.

Add the following after the explanation of Section J:

Section L. Fabrication

L-3. Welding Procedure. The use of alloy 5154 as a filler metal produces welds of greater strength and ductility than are generally obtained with alloy 4043. Welds in highly restricted locations may sometimes show cracking when 5154 filler metal is used. If this condition cannot be eliminated by adjustments of procedure or fixtures, the cracking tendency can be reduced by using 4043 filler metal. The allowable stresses given in Section J can be applied safely to welds made with either alloy.

L-4. Qualification of Welding Procedure and Welding Operator. The minimum requirement of 24 kips per sq in. for tensile strength of a reduced-section specimen used in a welding qualification test is the same as the minimum requirement in the ASME Boiler and Pressure Vessel Code.[30]

30. "Qualification Standard for Welding Procedures, Welders, and Welding Operators," ASME Boiler and Pressure Vessel Code Section IX, 1953, paragraph QN-6, p. 14.

Respectfully submitted,

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Discussion of
"NON-UNIFORM TORSION OF PLATE GIRDERS"

by Gerald G. Kubo, Bruce G. Johnston,
and William J. Eney
(Proc. Paper 449)

GERALD G. KUBO,[1] A. M. ASCE, and BRUCE G. JOHNSTON,[2] and WILLIAM J. ENEY,[3] M. ASCE.—Mr. Matteson is to be commended for his searching analysis of this paper. He has brought out several points which at first glance appear to be at variance with the writers' statements. However, it will be noted that much of the seeming difference can be attributed either to the vagaries of interpretation or of abridgement. In summarizing the material in the original report for publication, some of the background material had to be omitted.

The specimens tested and analyzed in this phase of the torsion program were shallow, relatively short plate girders without web stiffeners. In plate girders subjected to torsion the presence of properly-designed vertical web stiffeners would insure the maintenance of the original cross-section, thereby eliminating the possibility of web deformation. As Mr. Matteson points out, this aspect of the torsion problem requires further study.

Each specimen was divided into two symetrical half-spans with complete restraint against longitudinal warping at the plane of symmetry. In order to conform to the applicable theory as developed by Timoshenko as well as Goodier and Barton, it was necessary to have at the outer ends means whereby torsional moment could be resisted without restraint of longitudinal warping. The flange lug arrangement at diametrically opposite corners was a practical compromise, as noted. On the other hand, to follow the discusser's suggestion that "the flanges should be connected to the end support plates in some semi-rigid manner" would be a step in the wrong direction.

The writers are in agreement with the comments made by Mr. Matteson relative to the "finite beam," the "long beam" and the actual loading conditions. Certain basic differences in boundary conditions preclude the use of either solution as a direct comparison with test results. It was therefore necessary to utilize selected portions of the available theories. Apparently the intent of the writers in using the "finite beam" solution in the analysis of the angular twist in Figure 9 was misinterpreted. As explained, the finite beam solution was used to indicate that the flange reduction angle α as measured was of the same order of magnitude as that calculated for intermediate sections other than at the ends. It is agreed that conclusion No. 5 should have been so qualified.

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Mr. Matteson advances a logical explanation for the observed divergence from the expected pattern of the flange angle of twist at the free end. However, the essential difference between the discusser's suggested procedure and that used by the authors in Computation "H" appears to be one of degree. In the absence of a definite value for the unit angle of twist ψ at the end, the writers took the simple and conservative value of zero.

The secondary system of longitudinal normal stresses which develop in wide plates under torsion has also been mentioned by the discusser. This effect, explained by Timoshenko (8) has been extended to plate girders under uniform torsion by Chang and Johnston (4). The same specimens used in these tests had previously been subjected to uniform torsion loading. Chang and Johnston had concluded that "these longitudinal normal stresses are only important in estimating the ultimate torsion carrying capacity. In the useful load range (lower elastic range) their effect can be ignored." The end support detail at one end was arranged so that there was no restraint of the tendency of the twisted specimen to shorten. Although the pattern of these longitudinal stresses will be different in the non-uniform torsion case, the relatively low torque range used indicated that this effect would be negligibly small.

The writers are indebted to Mr. Matteson for his thoughtful suggestions which have contributed much to the value of the paper.

CORRECTIONS.—

1) page 449-3
   Eq. (4); coefficient of second term on right hand side should be 2/3 instead of 1/3.

2) page 449-3 last line
   spelling of word "longitudinal"

3) page 449-12
   In the second paragraph the modification noted (2) should read as follows:

   "2. The three equations based on boundary conditions required to evaluate the arbitrary constants were set up in terms of all the real and imaginary parts. Only after the three sets of arbitrary constants had been determined were the imaginary parts discarded on the basis of physical considerations."

4) page 8
   first sentence after "Test Set-up" the work "predicted" should read "predicated."

Discussion of
"SAFETY AND THE PROBABILITY
OF STRUCTURAL FAILURE"

by A. M. Freudenthal
(Proc. Paper 468)

A. M. FREUDENTHAL,[1] M. ASCE.—The discussion of the paper represents a very significant contribution to its value by the comments made, the questions raised and the detailed and painstaking re-evaluation of certain of the author's concepts. To an author there can hardly be a more gratifying response to a paper than a discussion carried on at this level.

Mr. Chibaro is certainly correct in assuming that we are still a long way from the day when the concept of "factor of safety" will be supplanted by the concept of "probability of failure." It is not the purpose of this paper even to advocate at present such a radical change in the engineering approach to structural design and analysis. Enough will be gained if the profession is gradually reconciled to the fact that the concept of "factor of safety" is meaningless unless it is supplemented by the specification of a "probability of failure" associated with any specific value of it, and that therefore reasonably efficient design, even the most careful and conservative one, tacitly implies an accepted risk of failure. The difference between the "safe" and the "unsafe" design is in the degree of risk considered acceptable, not in the delusion that such risk can be completely eliminated.

Mr. Chibaro's question concerning the ultimate value of probability considerations in the presence of unavoidable and significant non-random causes of variation has no satisfactory answer, if probability considerations are considered as an alternative to common-sense rather than as a complement to it, which should reduce the range of possible error in evaluating the contribution of random effects, and thus provide a better guide to correct guessing concerning the effects of the mathematically untractable non-random variations.

The author regrets that he has apparently not quite succeeded in clarifying all terms used in the paper. Some of the terms, the meaning of which is questioned by Mr. Chibaro have, however, acquired such definite connotations in the field of "plastic" or "limit-design" that they have been considered by the author as self-explanatory; this refers to the concept of "shake down load" (the maximum movable load pattern for which a condition of elastic-plastic equilibrium is possible), the "plastic collapse load" (the minimum intensity of a steady load pattern under which the system is transformed into an unstable kinematical chain) and the "elastic limit load" (the maximum intensity of a steady load pattern under which all deformations are reversible). With respect to the distinction between unserviceability and failure, however, it seems that the definition of these terms, to which section I.1. has been devoted, is still not sufficiently clarified. "Failure" and "unserviceability" are neither synonyms nor are they neighboring conditions. When serviceability

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ends because certain service requirements, such as a limiting maximum elastic deformation of a metal structure, or the absence of visible cracks in the tension zone of a reinforced concrete beam, are no longer fulfilled, the structure is usually still very far removed from conditions of designated failure or of actual collapse. In fact, the ratio between the intensities of a certain standard load pattern associated with certain probabilities of collapse and of unserviceability can be considered as an "overload factor" characteristic of the structure; this factor will be the higher, the larger the difference between the two load intensities, and will depend on the probabilities selected as acceptable. These probabilities may be equal for both conditions, or the acceptable probability of failure may have to be reduced against that of unserviceability because of the more serious consequences of the former, with a consequent reduction of the "overload factor;" in either case this factor is not represented by a single figure but by a probability function, specifying the probability of the structure to be able to withstand a certain ratio of "overload" in terms of the unserviceability load, without failure or collapse. The "overload factor" thus represents a characteristic that appears suitable for rating of existing structures.

Messrs. Lawrence and Corso have made a valuable contribution to the paper by proving independently that the author's procedure for the evaluation of the probability of failure in the case of normal or logarithmic-normal distributions of S and R can be improved and simplified by using the fact that, in this case, the distributions of $r = (R-S)$ or $\log r = \log (R/S)$ are also normal or logarithmic-normal. In the case of extreme value distribution no such simplification is necessary, since in this case the distribution functions are integrable in closed form and the values of P_F are obtained directly by multiplication of the respective cumulative probabilities, as indicated in eq. (29).

Both Mr. Lawrence and Mr. Julian are correct in pointing out that any meridional section of the surface of revolution shown in Fig. 10 should be normalized before being used for the evaluation of probabilities of failure; the slight discrepancies, in the discussed example, between Mr. Lawrence's and Mr. Julian's computed figures for P_F on the one hand and the author's figures on the other are due to this omission.

Mr. Corso has made a most interesting contribution by actually applying the principles outlined in the paper to the specific problem of the safety of a cluster of piles driven to rock. While it is true that the conclusions reached on the basis of this analysis are not new to experienced foundation engineers, the particular value of Mr. Corso's analysis is in its illustration of the fact that the conclusions can be derived by even a relatively inexperienced engineer on the basis of an objective analysis and that the results of this analysis provide numerical values of the safety factor or orders of magnitudes of the probability of failure for different pile arrangements, which permit a rational comparison of various arrangements without recurrence to "subjective experience."

The author does, however, not intend to create the impression as if, in this way, engineering experience and common sense could be dispensed with, but rather wishes to point to Mr. Corso's analysis as an illustration of how the safety analysis can provide a guide to common sense engineering decisions similar to that which structural analysis provides at a different stage of design. While most engineers to-day prefer to <u>compute</u> the maximum stresses expected in a certain structural part rather than to <u>guess</u> at their magnitude (even if they have sufficient experience to guess correctly), the same is not yet true with respect to the safety of the same part, for the estimate of which

guessing on the basis of personal experience and "intuition" is still the accepted procedure. By his analytical treatment of a problem of soil mechanics Mr. Corso has illustrated a rational alternative to this procedure in a field like soil mechanics in which most of the engineering decisions are, by neccessity, based on guesses rather than on rigorous analysis.

Mr. Julian has presented a very instructive and useful extension of the author's eq. (22) in the form of two tables, one for the rapid estimate of the required minimum factor of safety and the associated probability of failure, the other giving the results of such estimate under specific assumptions concerning the ratio of relative dispersion of S and R. In his general comments he succeeds, on the basis of his wide professional experience, to emphasize and reformulate several of the author's statements in a manner which clarifies their meaning for the professional engineer who is less interested in the methods of safety analysis themselves, than in the practical implications of the results. Mr. Julian's discussion of these implications contributes greatly to the understanding of the paper.

Concerning some of Mr. Julian's more specific comments, he is only partly right in disagreeing with the author's obviously too general statement about the effect of relaxation of prestress on the failure load of prestressed concrete beams: while such relaxation has indeed no effect on the failure load of under-reinforced beams failing primarily by yielding or fracture of the reinforcement, it actually reduces the failure load of over-reinforced beams failing primarily by crushing of the concrete. With respect to the question of using the term "probability" as a synonym for "relative frequency," the author believes to be on firm ground: he has been careful to use "probability" only with respect to continuous and, therefore, theoretical curves, reserving "frequency" for representation of actual observations. This usage is consistent with the frequency definition of probability, according to which a probability can only be the stochastic limit of an observed frequency. Hence, when speaking about continuous functions which, by their very nature, can not be directly observed but represent the "stochastic limit" fitted to the necessarily discontinuous observations, the term "probability" alone should be used; "frequency" can refer exclusively to observations, represented by a (discontinuous) histogram or by a sequence of points.

Mr. Gierasch stresses the principal advantage gained by a rational safety analysis, which is the possibility of attaining a balanced strength of all structural members, with the resulting optimal compromise between safety or probability of failure and economy, a compromise not based on intuition or "engineering judgment" but on a consciously accepted risk of failure that can be numerically specified and therefore compared with other risks, thus providing a measure against which the economic advantages of increasing or reducing this risk can be rationally assessed, as indicated in Section IV.11.

Discussion of
"THE VIBRATION OF STEEL STACKS"

by Walter L. Dickey and Glenn B. Woodruff
(Proc. Paper 540)

WALTER L. DICKEY[1] and GLENN B. WOODRUFF,[2] M. ASCE.—The authors appreciate the contributions of the several discussors who have done much toward formulating the questions which must be answered before any sound specification for the design of steel stacks can be prepared.

Professor Sherlock has pointed out the authors' failure to consider the effects of turbulence at high wind velocities and the probable frequency of comparatively non-turbulent winds of high velocity. Turbulence will interfere with the uniformity of vortex discharge and at least generally exists with high velocities. The effect of this turbulence may be termed wind efficiency (Ew), the numerical value of which decreases with increasing velocity and is also affected by the terrain. A low wind efficiency is the most plausible explanation that the amplitudes in the case described by Mr. Jenney were not serious.

A considerable part of the discussion hinged on the Strouhal Number. In an investigation at Stanford University under the direction of Prof. E. G. Reid, it appears that above the critical Reynolds number, the amplitudes increase as the square of the velocity except as affected by changes in damping and wind efficiency. These tests gave equal amplitudes at stagnation pressures of 5.25 and 13.65 psf (R = 465,000 and 685,000 respectively) indicating that the ratio of 1.71 and 0.66 assumed by the authors for C_L below and above the critical Reynolds number is approximately correct. Professor Farquharson quotes Landweber as $C_L = 3.8$ (below Reynolds critical) or over twice the value given by reference (8).

Mr. Jenney suggests, until firm design criteria becomes available, the use of welded steel stacks should be avoided in favor of reinforced concrete construction. Especially when area requirements are such that the stacks be placed on top of building steel, the latter are considerably more expensive. No cases of vibrations of lined stacks with amplitudes sufficient to produce excessive stresses were mentioned by the discussors.

Mr. Pirok questions the values for C_D and the assumption that welded stacks are more subject to vibration than riveted ones. The value of 0.35 for C_D was not intended as a design recommendation. For this purpose, the authors use the recommendations of A.C.I. Committee 505 for ordinary exposures, and increase the specified pressures in locations subject to hurricane. Mr. Pirok offers additional explanations of excessive vibrations in unlined welded steel stacks.

Mr. Pirok's statement of the relation between ovalling and cantilever vibrations deserves further investigation. The authors have no comments other

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than they believe ovalling is generally a forced vibration, the disturbing force being the vortices discharged at a frequency controlled by the natural frequency of the stack.

The authors agree with Mr. Pirok's opinion of external damping devices. Such devices may be economical in the case of completed stacks which are subject to excessive vibrations.

Mr. Vincent furnished considerable detail as to the nature of the disturbing force. He also adds twice the amplitude to the effective diameter of the stack. In the usual case the relation of the amplitude to the diameter is so small that the correction therefor is minor compared to the other unknowns. Mr. Vincent questions the value of π/δ for the magnification factor. The authors believe this value high and a factor of $1/\delta$ corresponds better with available observations.

Messrs. Vincent and Pagon furnish additional observations and data as to the forces on leeward stacks. Wind tunnel tests at values of R above the critical are needed.

Professor Farquharson adds considerable information as to critical values of R and suggests the addition of "spoilers" which would reduce the intensity of the lateral forces. Both these matters were investigated in the Stanford tests above-mentioned. These tests indicated an effective system of spoilers but none that appeared more economical than designing the stacks to allow for the dynamic forces.

In the Stanford tests, C_D dropped from 0.9 at R = 250,000 to 0.3 at R = 600,000. The maximum response at Reynolds subcritical occurred at R = 425,000 with a Strouhal Number of 0.34.

The economical design of steel stacks and circular pressure vessels is important to industry. The authors trust that means will be found to explore the many unknowns that now exist.

CORRECTIONS.—

Eq (1) Change 0.0019 to 0.00119

Eq (8) Change $\sqrt{\frac{gWx}{\Sigma Wx^2}}$ to $\sqrt{\frac{g\Sigma WX}{\Sigma WX^2}}$

Discussion of
"ULTIMATE SLOPES AND DEFLECTIONS—
A BRIEF FOR LIMIT DESIGN"

by George C. Ernst

A. DIWAN,[1] A. M. ASCE.—In connection with the theory of "Limit Design" it is usually assumed that the relation between the moment M acting at any section and the unit rotation ψ produced there is as shown in figure (8) type (1). This relation however is not suitable for producing any necessary plastic hinge unless the B.M. is constant throughout some length of the beam, sufficient to accumulate the required rotation. In the case of a concentrated load where the bending moment reaches its max value at one section only no such plastic hinge can possibly develop (since the length of the yielding part is theoretically zero). It is therefore necessary to consider as an idealized $M - \psi$ relation, the second curve-type (2)—figure (8), in which the plastic range starts at a moment M_o called the yield moment and continues with a slight increase in the bending moment until the ultimate value M_u with an ultimate unit rotation ψ_u where final collapse occurs. The plastic portion in the frame will then extend throughout the length where the moment exceeds the yield value M_o. This length however may not be sufficient to accumulate the necessary plastic rotation at the section, in which case, at least theortically, further plastic rotations, at other sections will not possibly develop .

Such a distinction between the yield moment M_o and the ultimate moment M_u, although not necessary when dealing with the state of final collapse, is important when computing the slopes and deflections prior to that state. No such distinction is made by the author when using the unit rotation diagram in place of the (M/EI) diagram.

Indeed, the use of the unit rotation diagram, besides lacking most of the advantages of the $(\frac{M}{EI})$ diagram, seems to have no merit. As a matter of fact the $(\frac{M}{EI})$ diagram possesses the following advantages:

1) The conditions of equilibrium at any joint are easily seen, especially when the members framing into the joint have different cross-sections. The unit rotations there will not be equal, and will not all reach the yield value ψ_o.
2) The bending moment is allowed to reach the ultimate value M_u at the sections where the plastic hinges develop, while it only reaches the yield value M_o at the section on the verge of plastic range. When using the unit rotation diagram, the author takes the same ψ_o value for both types of sections.
3) The equilibrium conditions necessary to draw the B.M.D. due to any loading system prior to final collapse are directly available, and the ultimate value of the loads may be readily computed.

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Under the title "Revised Moment–Area Theorems," the following theorem may be added.

"The component of the translation at any point in the frame in any direction is equal to the sum of the moments at that point for all the elastic weights of the $(\frac{M}{EI})$ diagram, and the rotations at the end support and at the plastic hinge—to one side of the point—all considered as forces acting parallel to the required direction of translation."

The summation may start at any intermediate point provided the rotation and translation there are known and are appropriately included in the summation.

This theorem is essential for the analysis of the frames, and makes it possible to arrive at the final solution without the need of solving several simultaneous equations as presented by the author. This is shown later by several examples.

In general, the available conditions of static and elastic equilibrium will be sufficient to determine both the B.M.D. and the magnitudes of the plastic rotations, due to any type of loading on the frame prior to final collapse. By elastic equilibrium is meant the equilibrium of the elastic weights.

Plastic Properties of R.C. Sections

An idealized stress-strain relation for concrete is shown in fig. (1) The curve is taken as a second degree patabola with a vertical axis at the maximum concrete stress f'_c corresponding to the strain ϵ'_c .

The ultimate strain ϵ_u is assumed to be $1.5\,\epsilon'_c$ with $f_u = 0.75\,f'_c$. Experiments show that the strain ϵ'_c does not vary much with various concrete mixes, and although it may be taken as constant it is not necessary to do so. With such an idealized curve the only variables governing the inelastic behaviour of any concrete mix are f'_c and ϵ'_c.

For under-reinforced sections, the steel will reach the yield stress f_{yp} first, and the strain therein will be:

$$\varepsilon'_s = \frac{f_{YP}}{E_s}$$

With the steel reaching the yield stress f_{yp}, the section will become on the verge of plastic yield. For a rectangular section, the position of the neutral axis at this stage will be given by:

$$p \cdot \frac{f_{YP}}{f'_c} = \frac{k^2}{a(1-k)}\left[1 - \frac{k}{3a(1-k)}\right] \quad \ldots\ (1)$$

where ;

$$a = \frac{\varepsilon'_c}{\varepsilon'_s}$$

The rotation (ψ_o) per unit length at this instant will be:

$$\psi_o = \frac{\varepsilon'_s}{d(1-k)} = \frac{C_3}{d} \qquad (2)$$

The moment arm (J) will be:

$$J = d\left\{1 - \frac{k}{4}\left[\frac{4a(1-k)-k}{3a(1-k)-k}\right]\right\} \qquad (3)$$

The moment of resistance of the section then will be:

$$M_o = p\, f_{YP}\, bdJ$$

or

$$M_o = C_2\, f'_c\, bd^2 \qquad (4)$$

The moment M_o and the rotation (ψ_o) will be called the yield moment and yield rotation, respectively.

Up to (M_o), the relation between M and ψ may be assumed as linear, with a section stiffness (EI) such that:

$$EI = \frac{M_o}{\psi_o} = C_4\, f'_c\, bd^3 \qquad (5)$$

The constants K, C_3, C_2 and C_4 are given in figures 1, 2, 3, and 4 respectively for the values a = 2.4, 1.8, 1.45 and 1.20. For $\epsilon'_c = 0.0024$ and $E_S = 30 \times 10^3$ ksi, these "a" values will represent the cases f_{yp} = 30, 40, 50, and 60 ksi respectively. It can however be seen that M_o is not affected much by the variation in the parameter "a."

As the moment on the section exceeds M_o, the steel stress f_{yp} will remain constant while the neutral axis will shift upwards until finally the stress distribution over the section will be as shown in figure (5) when the moment reaches the ultimate value M_u.

The total compression C and tension T will be:

$$C = 0.75\, kbdf'_c$$

$$T = p.\, bd\, f_{YP}$$

Thus:

$$K = \frac{4}{3}\, p\, \frac{f_{YP}}{f'_c} \qquad (6)$$

and;

$$J = (1 - 0.416\, k)\, d \qquad (7)$$

The ultimate moment M_u, will be,

$$M_u = C_6 \; f'_c \; bd^2 \qquad (8)$$

It is seen that K, J & M_u will not depend on the strain ϵ'_c

The ultimate rotation ψ_u will then be,

$$\psi_u = \frac{1.125 \;\; \epsilon'_c \cdot f'_c}{p \; f_{yp} \; d} = \frac{C_5}{d} \quad \ldots\ldots (9)$$

Fig. (5) gives the constant C_6 for all values of ϵ'_c and the constant C_5 for $\epsilon'_c = 0.0024$ and 0.0020.

Analysis of a Single Bay Frame

For a yield or plastic hinge to develop at any section the moment there should reach the ultimate value (M_u). Total collapse will take place when four plastic hinges form, while local collapse will occur when only three hinges develop in any one straight part of the frame. The positions of these plastic hinges for any case of loading at collapse can be determined by trial[2] or by relaxation.[3] We are however dealing with the state prior to final collapse when the last yield hinge has not yet developed, but is on the verge of doing so, with the moment there reaching the yield value M_o. At which section the plastic hinge will develop last is best determined by proper judgement, keeping in mind that the elastic weights corresponding to the rotations ϕ at the plastic hinges must have the same sign as the ultimate moments (M_u) there. Any violation to this rule means a wrong assumption in the location of the plastic hinge. It will be seen that for any loading there will be statical and elastic conditions sufficient to determine the B.M.D. and the values of the rotations at the plastic hinges.

Consider the frame in fig. (6) in which $f'_c = 4.0$ ksi $\epsilon'_c = 0.0024$, $f_{yp} = 40$ ksi, and $E_s = 30{,}000$ ksi, thus a = 1.80.

The values EI, $\overline{L}$, M_o, M_u, ψ_o & ψ_u for the three members AB, BC and CD in ft and kip units are tabulated.

As a first example, consider loading (1), fig. (6.b) producing local collapse in the girder BC. Prior to collapse there will be two plastic hinges, the first under the load P and the second at either B or C in the column BA or CD respectively. In fig. (6.b') it is assumed that the hinges develop at F and C, thus the moment will reach the ultimate values there, while it only attains the yield value (M_o) at B. We notice that the governing moments at B and C are those corresponding to the columns since they are less than those for the girder. However, the moments at A and D are still unknown.

If $M^A = X$, then from statics $M^D = X = 45$.

Since the horizontal translations at B and C are equal, then,

2. "Limit Design of Beams & Frames" by Greenberg & Prager Trans. ASCE. vol. 117 p. 447.
3. "Design of frames by relaxation of yield-hinges" by Morley English. Trans. ASCE. vol. 119 p. 1143.

$$(2x - 170)\,16.4 = [125 - 2(x+45)]\,21.4$$

from which $X = 47$

$\therefore M^A = 47$ ft. kips, $M^D = 92$ ft kip.

We have two more conditions of elastic equilibrium; or equilibrium of elastic weights, from which the unknown concentrated elastic weights at F and C corresponding to the rotations ϕ^F and ϕ^C at the plastic hinges there can be readily computed; thus:

1. $\Sigma_A^C M^C = 0$

$\therefore \phi^F = 0.0108$ radians.

2. $\Sigma_D^B M^B = 0$ (with $\phi^F = 0.0108$)

$\therefore \phi^C = 0.0038$

Should however the plastic hinge be assumed at B instead of C, the rotation ϕ^F would have been:

$$\phi^F = -0.00397$$

The negative sign, meaning that it is not consistent with the ultimate moment M_u there, indicates that this assumption for the location of plastic hinges is impossible.

For a second example, consider loading (2), fig (6-c) producing total collapse in the frame. Prior to collapse three plastic hinges will have fully developed with the moment reaching the yield value M_o at a fourth section. Fig. (6-c') shows the assumed state prior to collapse, with plastic hinges at F, C, and D where the moments have reached the ultimate value M_u and with the moment at A just reaching the yield M_o.

The only unknown moment is that at B which by simple statics is found to be 24.

It is now seen that the problem contains three unknown elastic weights. Since there are three conditions of elastic equilibrium the problem is determinate. For the plastic rotation ϕ^F, assuming all elastic weights to act vertically;

$$\Sigma_A^C M^C = 0 \qquad \therefore \phi^F = 0.00885$$

Next, with all elastic weights acting horizontally;

$$\Sigma_A^D M^D = 0 \qquad \therefore \phi^C = 0.01120$$

Finally, assuming the elastic weights to act horizontally, and from the equality of the horizontal transations at B and C.

$$\Sigma_A^B M^B = \Sigma_D^C M^C$$

from which

$$\phi^D = 0.00425 \quad \text{radians}.$$

Multiple—Bay Frames

The analysis may be extended to the case of continuous multiple-bay frames prior to total or local collapse. An example is shown in fig. (7), where the final collapse is going to be local in girder AB. Since however there will be no plastic hinge developing in the bay EBCF up to the state of final collapse, this part may be considered as remaining within the elastic range. The final solution can be simplified by reducing the continuous frame to a single bay virtual system with the part (EBCF) replaced by an equivalent elastic system,[4] as shown in figure (7.b). The elastic constants for the equivalent symbolic system will be

$$\bar{S} = .0000185$$

$$\bar{Y} = -4.7'$$

$$I_x = 0.0038$$

Prior to collapse the B.M.D. will be as shown in figure (7.C), with the plastic hinges fully developing in AB, at J and B, and with the moment at A reaching the yield value for column AD. If the moment at D is equal to X, then by simple statics the moment at the centroin O of the symbolic system will be (242 - 0.235 X).

From the equality of the horizontal translations at A and B, we get:

$$\Delta^A = \frac{20}{6}(170 - 2x)(.000164)$$

$$\Delta^B = 4.7(.235x - 242)(.0000185) + \left(\frac{170 + x}{20}\right)(.0038)$$

From which

$$X = 63 \quad \text{Kips. ft.}$$

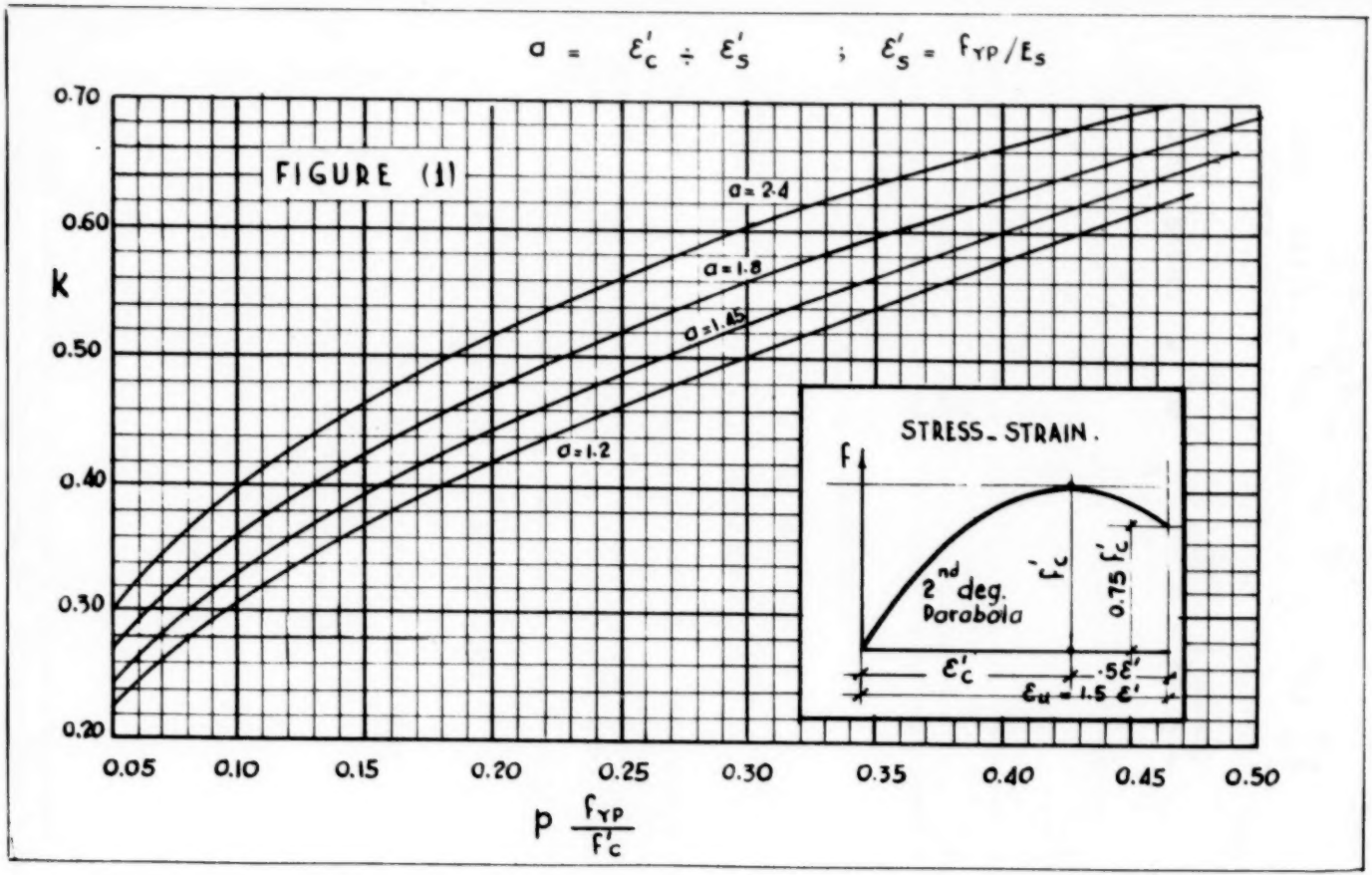
4. "Equivalent Elastic Systems in The Analysis of Continuous Structures" L. Beaufoy & A. Diwan. Conc. & Const. Eng. 1950. See also discussion by A. Diwan, of Proceedings—Sep. No. 586.

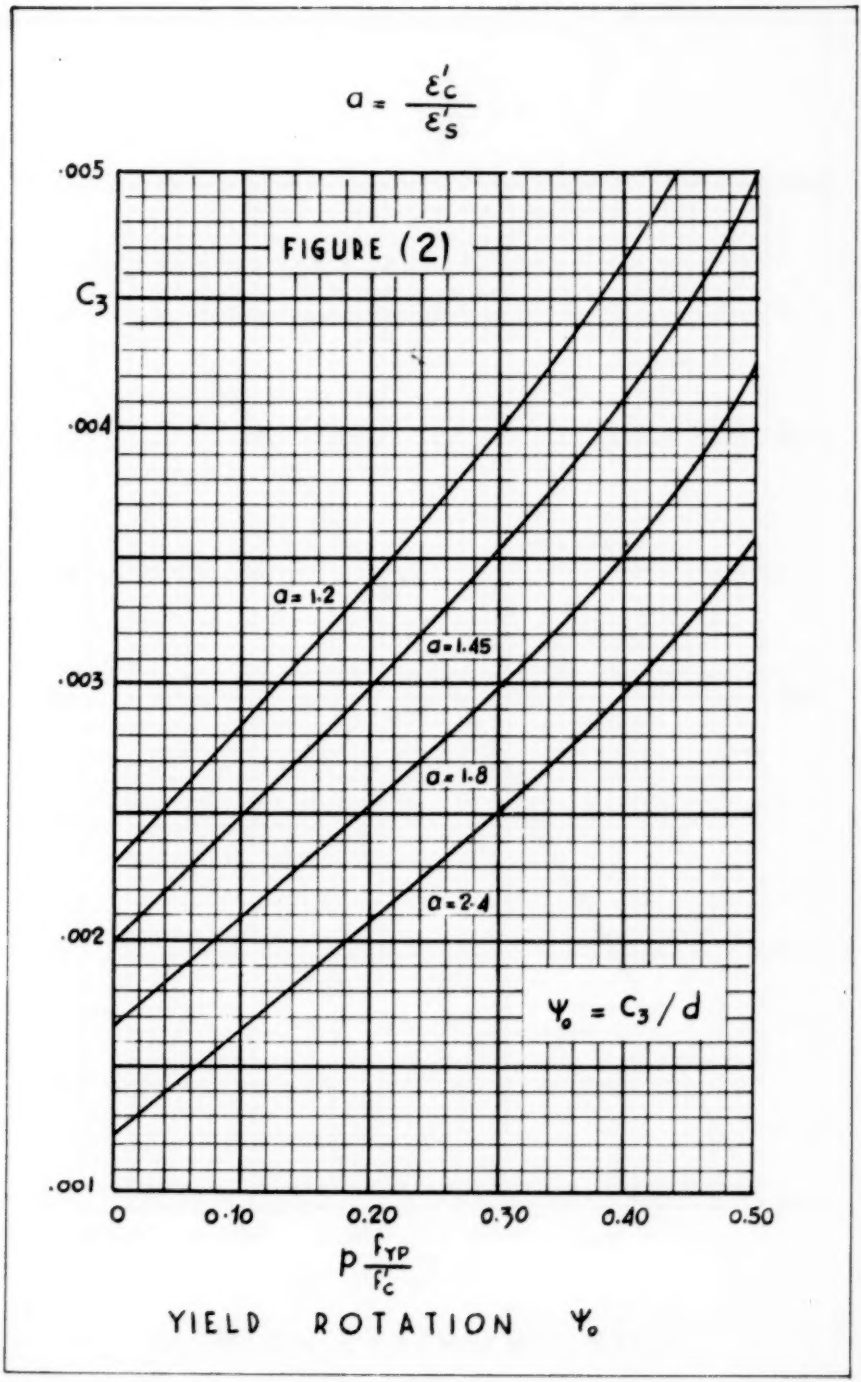
Then:

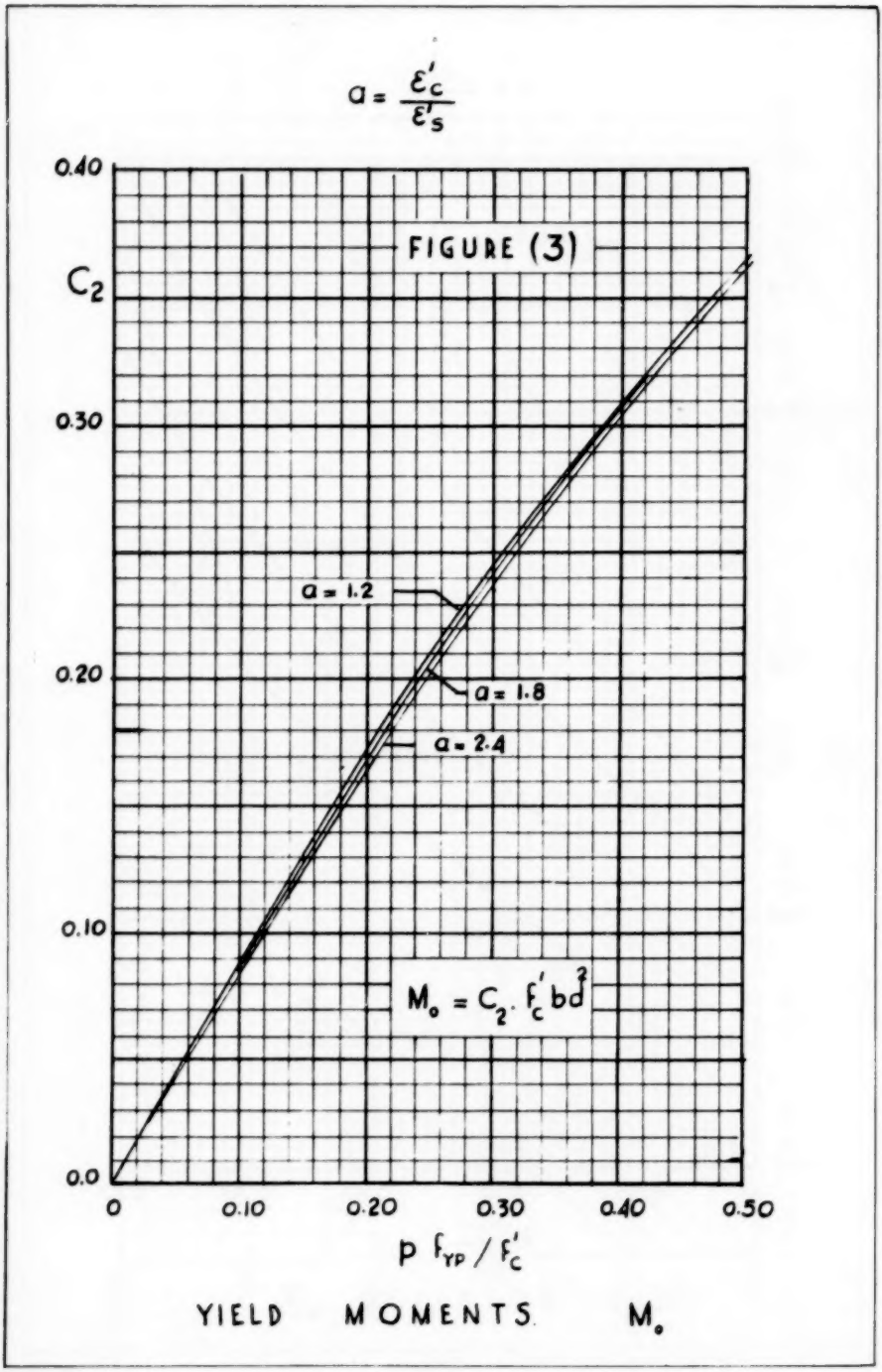
$$\Sigma_D^B M^B = 0 \qquad \therefore \quad \phi^J = .01335 \text{ rad.}$$

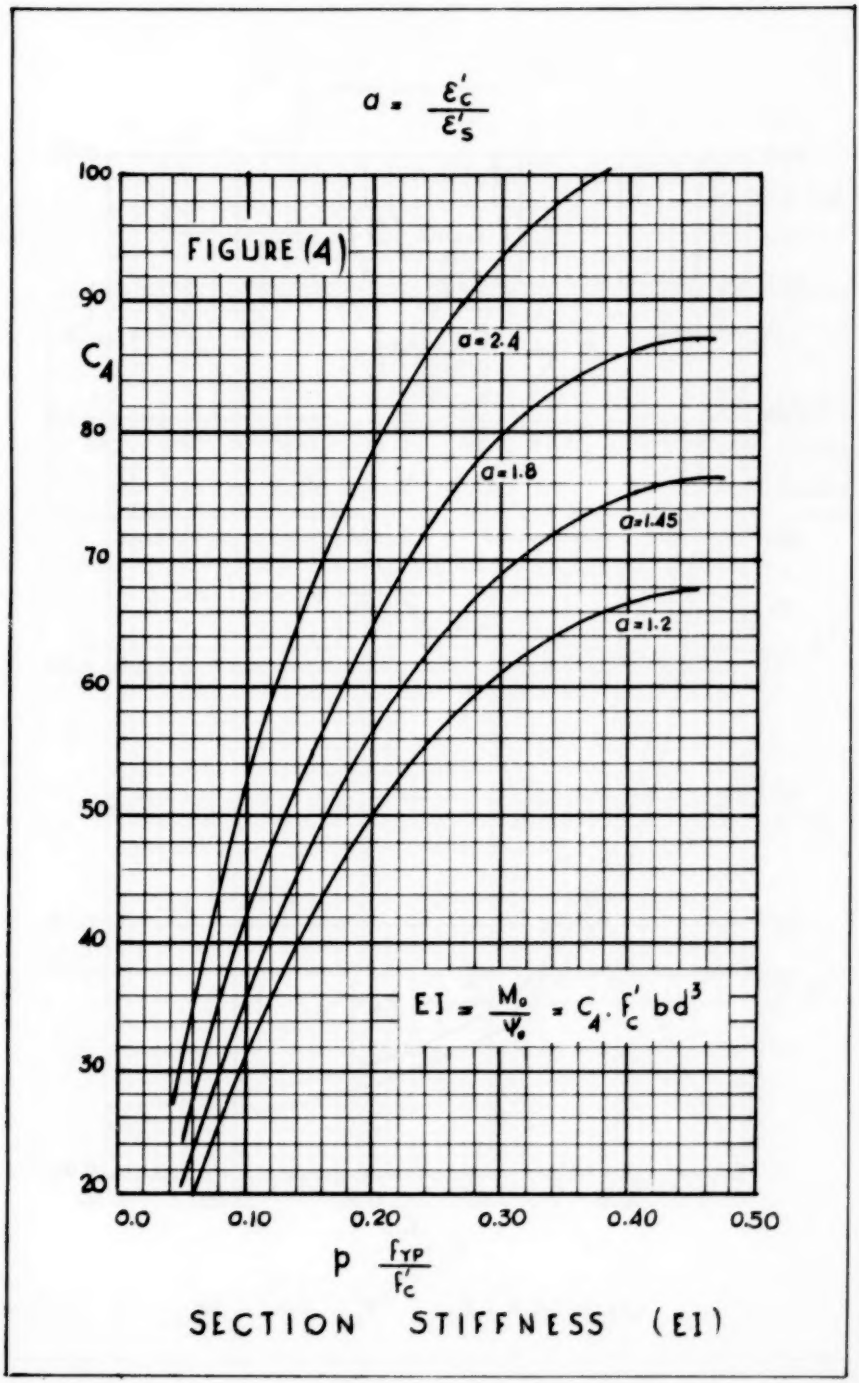
and

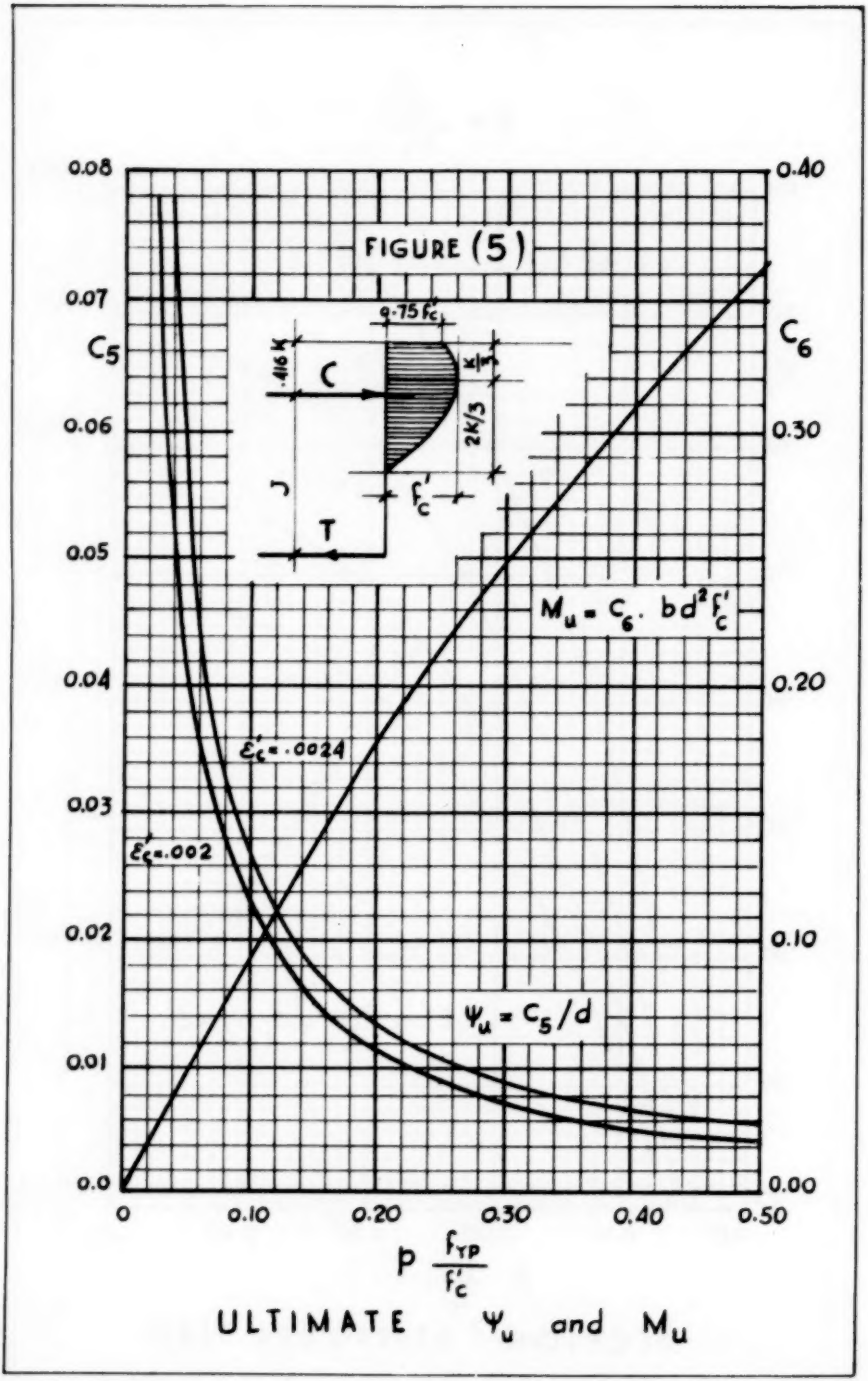
$$\Sigma_O^A M^A = 0 \qquad \therefore \quad \phi^B = .00288 \text{ rad.}$$


a = $\varepsilon'_c \div \varepsilon'_s$; $\varepsilon'_s = f_{YP}/E_s$
FIGURE (1)
a = 2.4
a = 1.8
a = 1.45
a = 1.2
k
0.70
0.60
0.50
0.40
0.30
0.20
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
p $\frac{f_{YP}}{f'_c}$
STRESS - STRAIN.
f
f'_c
0.75 f'_c
2nd deg. Parabola
ε'_c
.5ε'_c
ε_u = 1.5 ε'_c

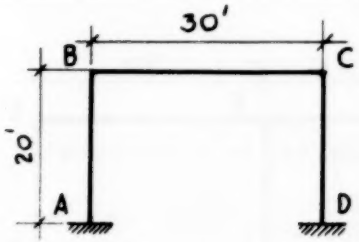

$a = \frac{\varepsilon'_c}{\varepsilon'_s}$
FIGURE (2)
C_3
.005
.004
.003
.002
.001
a = 1.2
a = 1.45
a = 1.8
a = 2.4
$\psi_o = C_3 / d$
0
0.10
0.20
0.30
0.40
0.50
$p \frac{f_{YP}}{f'_c}$
YIELD ROTATION ψ_o


$a = \frac{\varepsilon'_c}{\varepsilon'_s}$
FIGURE (3)
C_2
0.40
0.30
0.20
0.10
0.0
a = 1.2
a = 1.8
a = 2.4
$M_o = C_2 \cdot f'_c b d^2$
0
0.10
0.20
0.30
0.40
0.50
$p\, f_{yp} / f'_c$
YIELD MOMENTS M_o


$a = \frac{\varepsilon'_c}{\varepsilon'_s}$
FIGURE (4)
100
90
80
70
60
50
40
30
20
C_4
a=2.4
a=1.8
a=1.45
a=1.2
$EI = \frac{M_o}{\psi_o} = C_4 \cdot f'_c\, bd^3$
0.0
0.10
0.20
0.30
0.40
0.50
$p\ \frac{f_{YP}}{f'_c}$
SECTION STIFFNESS (EI)

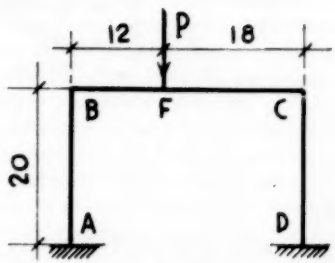


ULTIMATE Ψ_u and M_u

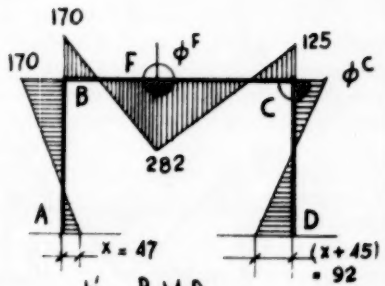


f'_c = 4 ksi
ε'_c = 0.0024
E_s = 30,000 ksi

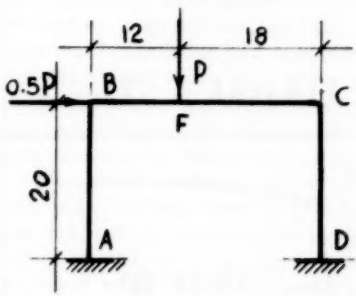
MEMBER	SECTION inch.	p	EI K.Ft^2	$\bar{L} = \frac{L}{EI}$ $(K.Ft)^{-1}$	M_o K. ft.	M_u K. ft.	ψ_o radians/inch	ψ_u radians/inch
AB	10 x 20	.015	122000	.000164	170	183	.000116	.0009
BC	10 x 30	.010	32×10^4	.000094	264	282	.000069	.0009
CD	10 x 20	.010	93500	.000214	117	125	.000103	.00135



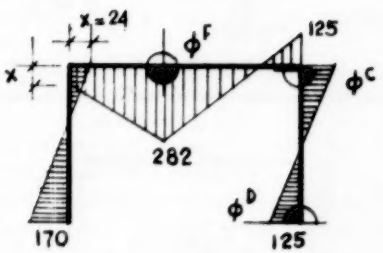
b. LOADING (1)



b' – B.M.D.

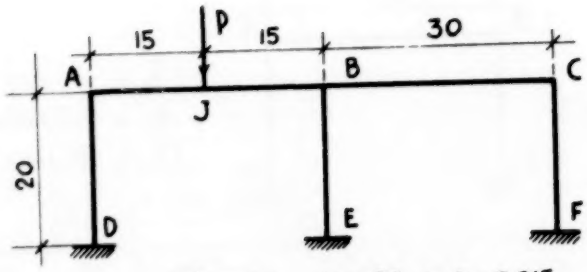


C. LOADING (2)



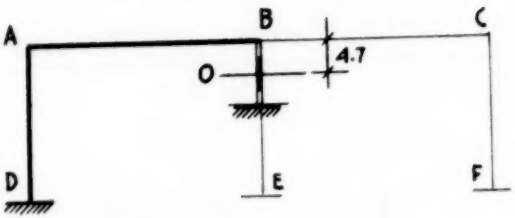
c'. B.M.D.

FIGURE (6)



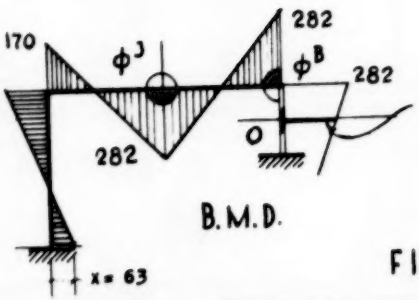
$f'_c = 4$ ksi
$\varepsilon'_c = .0024$

a - Columns 10 x 20 ; p = 0.015
Girders 10 x 30 ; p = 0.010



b -

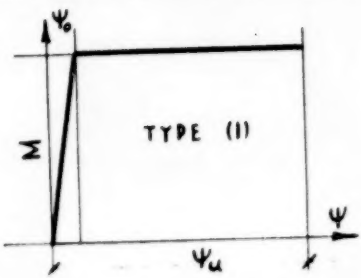
EQUIVALENT SYSTEM
$\bar{S} = .0000185$
$\bar{Y} = -4.7'$
$I_x = .00380$



$M^o = 242 - 0.235 \times$
$= 227$

C - B.M.D.

FIGURE (7)



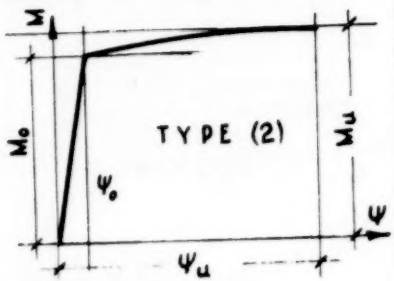


FIGURE (8)

Discussion of
"MULTIPLE SPAN GABLED FRAMES"

by John D. Griffiths
(Proc. Paper 586)

A. F. DIWAN,[1] A.M. ASCE.—It has always been realized that the main difficulty involved in the analysis of continuous structures by the classical methods lies in the necessity of preparing and solving as many simultaneous equations as the structure is statically indeterminate. Many attempts have therefore been made by structural engineers to develop more practical and simple methods of analysis in which no such simultaneous equations are needed. The most successful attempt is no doubt the relaxation method presented by Prof. H. Cross.(1) Ever since Cross published his work, further developments have been added to the original method, all of which tend to accelerate the divergence of the relaxation process, or even so to get a direct one step relaxation. The method presented by the author—An Analysis By Parts—in one of the modifications to the original method of Cross. Proceeding from one end of the frame, the joints are successively relaxed until we reach the other end when all joints will be balanced. Once any joint is unlocked, it remains unlocked and the modified fixed end moment and thrust and the rotation and translation stiffnesses at the next joint are calculated.

Almost all the joints have two degrees of freedom, and the necessary balancing displacement consists of a rotation θ and a horizontal translation Δ. In the suggested solution, the joint is first allowed to rotate until the moment acting there is balanced. The resulting force, together with that originally acting on the joint, are next balanced by allowing translation Δ with appropriate rotation θ such that no moments are to be developed at the joint. This is of course, a modification to the original method of Cross in which pure rotation and translation are separately produced at the so called "Neutral Point" for the joint.

In the author's paper, the simple unit considered is always one straight bar, except for CDE which is taken as one unit. This is rather lengthy, although it has no doubt caused simplicity in the procedure.

It is however more convenient, in the writer's opinion, to reduce the number of the simple units as much as possible, and thus reduce the number of the joints to be relaxed and at which the displacements θ and Δ are to be computed, without still having to solve any equations. For the given frame, only joints B and F need to be locked at the start, thus dividing the frame into three parts namely (BK & BJ), BDF, and (FH & FI).

These three units are added in series, while the parts in brackets are added together in parallel, to form one unit, each.

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The analysis of continuous frames, such as that dealt with by the author is much simplified by the concepts of equivalent elastic systems[2],[4] and stiffness factors[3] published by the writer, together with L. Beaufoy, some years ago. It is interesting to show the application of these methods to the solution of the given frame, shown in figure (1). Computations are all made by slide rule, and no equations are needed. The method is first explained as follows.

For any elastic member AB, fig. (2), polygonal or straight, with constant or variable cross section, the following terms are called the elastic constants:

$$\bar{S} = \int \frac{ds}{EI} = \int d\bar{s} \qquad \ldots (1)$$

$$I_x = \int y^2 \, d\bar{s} \qquad \ldots (2)$$

$$I_y = \int x^2 \, d\bar{s} \qquad \ldots (3)$$

$$I_{xy} = \int xy \, d\bar{s} \qquad \ldots (4)$$

C is the centroid of the elastic lengths $d\bar{s}$.

From these elastic constants, the rotation θ and the horizontal and vertical translations Δ and λ of the elastic centroid C due to a moment M, a horizontal thrust T and a vertical force V, acting there, will be:

$$\theta^C = M\bar{S} \qquad \ldots (5)$$

$$\Delta^C = T I_x - V I_{xy} \qquad \ldots (6)$$

$$\lambda^C = V I_y - T I_{xy} \qquad \ldots (7)$$

Positive moments and rotations are clockwise, and positive translations and forces are upwards and to the right.

The displacement of end A assumed as rigidly connected to C in terms of the displacement of C will then be:

$$\theta^A = \theta^C \qquad \ldots (8)$$

$$\Delta^A = \Delta^C - \bar{y}\,\theta^C \qquad \ldots (9)$$

$$\lambda^A = \lambda^C + \bar{x}\,\theta^C \qquad \ldots (10)$$

On the other hand, the moment M and forces T and V necessary to act at C to produce rotation θ and translations Δ and λ there will be:

$$M = 1/\bar{S} \qquad \ldots (11)$$

$$T = \Delta' / I'_x \qquad \ldots (12)$$

$$V = \lambda' / I'_y \qquad \ldots (13)$$

in which :

$$I'_x = I_x - I^2_{xy} / I_y \quad \ldots (14)$$

$$I'_y = I_y - I^2_{xy} / I_x \quad \ldots (15)$$

$$\Delta' = \Delta + \lambda . I_{xy} / I_y \quad \ldots (16)$$

$$\lambda' = \lambda + \Delta . I_{xy} / I_x \quad \ldots (17)$$

The term "Stiffness factors" for a Joint denotes the moment, horizontal and vertical forces necessary to produce unit rotation, horizontal or vertical translation there. These factors are given thus:

M_θ , T_θ & V_θ for unit rotation θ

M_Δ , T_Δ & V_Δ for unit translation Δ

M_λ , T_λ & V_λ for unit translation λ

From the reciprocal relation:

$$M_\Delta = T_\theta \quad \ldots (18)$$

$$M_\lambda = V_\theta \quad \ldots (19)$$

$$T_\lambda = V_\Delta \quad \ldots (20)$$

These rotation and translation stiffnesses for Joint A of the elastic member AB will therefore be:

$$T_\Delta = 1 / I'_x \quad \ldots (21)$$

$$V_\lambda = 1 / I'_y \quad \ldots (22)$$

$$V_\Delta = T_\lambda = I_{xy} / I'_x I_y \quad \ldots (23)$$

$$M_\Delta = T_\theta = \bar{y} T_\Delta - \bar{x} V_\Delta \quad \ldots (24)$$

$$M_\lambda = V_\theta = \bar{y} T_\lambda - \bar{x} V_\lambda \quad \ldots (25)$$

$$M_\theta = \frac{1}{\bar{S}} + \bar{y} T_\theta - \bar{x} V_\theta \quad \ldots (26)$$

Applying this to the middle part BDF of the frame as shown in fig. (3); we get the following values for the elastic constants for the frame, and for the stiffness factors for end B assuming joint F to be fixed.

$\bar{S}$ = 2 (0.10 + 0.025) = 0.250

I_x = 3.0 = I'_x

I_y = 67.2 = I'_y

I_{xy} = 0

then :

$T_\Delta = 0.333$

$T_\theta = 4.0 = M_\Delta$

$V_\theta = -0.357$

$M_\theta = 60.57$

The carry-over moments to end F will be :

$M'_\Delta = -4.0$; $M'_\theta = -43.43$

On the other hand, if the stiffness factors for any joint, in which two or more elastic member intersect, are known, the elastic behaviour of the joint can be represented by that of a single elastic member connected to the joint, and called the equivalent elastic system.

The elastic constants for this symbolic system in terms of the stiffness factors for the joint are given in table (1), for the general case of a joint with three degrees of freedom, and the special cases of only two degrees of freedom.

Considering for example the two members FH and FI, intersecting at F, and added in parallel, the sum of their stiffnesses, with joint G unlocked will be: (see fig. 4)

$$M_\theta = 38 + 80 = 118$$

$$T_\theta = 2 - 7.5 = -5.5$$

$$T_\Delta = 1.33 + 0.94 = 2.27$$

Since only rotation and horizontal translation can take place at F, the elastic constants for the equivalent system will be:—see table (1)—

$$\bar{y} = -\frac{5.5}{2.27} = -2.43'$$

$$I_x = 1/2.27 = 0.44$$

$$\bar{S} = \frac{2.27}{118(2.27) - (5.5)^2} = 0.0095$$

If this symbolic system is now added in series to BDF as shown in figure (5), we get a new virtual elastic system for which the elastic constants will be:

$\bar{S} = 0.2595$ then :

$I_x = 5.35$ $I'_x = 5.20$

$I_y = 72.5$ $I'_y = 70.5$

$I_{xy} = -3.18$

The stiffness factors for the end B of this virtual system will be the real stiffnesses for that end with no external additional restraint applied any where to the right of B. The same values of course apply to end F of FDB due to symmetry.

These stiffnesses will be (see equations 21 - 26):

$$T_\Delta = 0.192$$
$$V_\Delta = -0.0085$$
$$M_\Delta = 2.41 = T_\theta$$
$$V_\theta = -0.450$$
$$M_\theta = 42.80$$

The displacement induced at the far joint F due to the unit translation or unit rotation taking place at B are obtained from the moment and thrust acting at the centroid 0 of the symbolic system:—see equations (5 - 10)—

Thus for unit translation Δ at B, the moment and thrust at 0 are:

$$T = 0.192$$
$$M = 2.41 + 0.192(2.43) - .0085(48)$$
$$= 2.47$$

The displacements at F will then be: (see equations 5-10)

$$\theta_\Delta = 2.47\ (0.0095) = 0.0235$$
$$\Delta_\Delta = 0.0235(2.43) + 0.192\ (0.44)$$
$$= 0.142$$

Similarly, for a unit rotation at B, the induced motion at F will be:

$$\theta_\theta = 0.257$$
$$\Delta_\theta = 1.68$$

The actual rotation and translation stiffnesses for joint B, with the three intersecting members, and with no restraint applied any where else in the frame can now be obtained, thus:

$$M_\theta = 38 + 80 + 42.8 = 160.8$$
$$T_\theta = 2 - 7.5 + 2.41 = -3.09$$
$$T_\Delta = 1.333 + 0.94 + 0.192 = 2.465$$

The elastic constants for the symbolic system equivalent to joints B and F in the full frame will be:

$$\bar{S} = 0.00638$$
$$I_x = 0.407$$
$$\bar{y} = -1.25$$

With these values determined, a direct one step relaxation can be obtained. The steps of the solution will be as follows:

1) With only joints B and F locked, evaluate moments and thrusts acting there, using any convenient method, say column analogy for BDF and ordinary moment distribution for BK.

Thus we get:

At B : $M = 581 - 2.6 = 578.4$ ft.k.

$T = 118.1 + 89.3 = 207.4$ K

At F : $M = 326$ ft. k.

$T = 31.9$ K.

2) Transferring these moments and thrusts to the elastic centroids of the joints we have:

For Joint B : $M = 207.4\,(1.25) + 578.4$

$= 837.6$ ft. k.

$T = 207.4$ K

Joint F : $M = 31.9\,(1.25) + 326$

$= 366$ ft. k.

$T = 31.9$ k.

3) Evaluate the balancing displacements at each joint using equations (5 - 10) and those consequently induced at the other joint using the coefficients

$$\theta_\theta = 0.257\,, \quad \Delta_\theta = 1.68$$
$$\theta_\Delta = 0.0235\,, \quad \Delta_\Delta = 0.142$$

These displacements are given in table (2).

4) The moments M and forces T, acting on the terminals at each joint are next computed as shown in tables (3) and (4). From these values, the moments and forces at any other section in the frame are easily obtained by simple statics.

The results obtained for the displacements θ and Δ, and for the moments M and thrusts T agree with those given by the author with a maximum difference of less than 0.5 per cent. We notice that the method is self checking, since the final moments and forces at any joint must sum up to zero.

Finally figure (6) shows the elastic constants for tapered members in which the inertia varies with d^3. From these the stiffness factors can be readily evaluated for both cases of totally fixed or hinged far end. In the later case the centroid lies at the hinge. It is hoped that the concepts of equivalent elastic systems, and stiffness factors will prove useful to all concerned with the design of continuous structures.

REFERENCES

1. "Continuous Frames of R. C." by H. Cross & N. Morgan.
2. "Equivalent Elastic Systems in The Analysis of Continuous Structures" by L. Beaufoy & A. Diwan. Conc. & Const. Eng. vols— 11-12. 1950.
3. "Analysis of Continuous Structures by The Stiffness Factors Method." by L. Beaufoy & A. Diwan. Quart. Journ. Mech. & Applied Math. Pt. 3, 1949.
4. "Analysis of Pin — Jointed Redundant Plane Frameworks Using Equivalent Elastic Systems" by L. Beaufoy and A. Diwan. Quart. Journ. Mech. & Applied Math. Pt. 1, 1950.

TABLE (1) Elastic Constants in Terms of of Stiffness Factors

Elastic Constants	General Case 3 degrees of freedom	Δ and θ only possible	λ and θ only possible
$\bar{S}$	$\frac{D_1}{D}$	$\frac{T_\Delta}{D_4}$	$\frac{V_\lambda}{D_5}$
$\bar{x}$	$\frac{D_2}{D_1}$	0	$-\frac{V_\theta}{V_\lambda}$
$\bar{y}$	$\frac{D_3}{D_1}$	$\frac{T_\theta}{T_\Delta}$	0
I_x	$\frac{V_\lambda}{D_1}$	$\frac{1}{T_\Delta}$	0
I_y	$\frac{T_\Delta}{D_1}$	0	$\frac{1}{V_\lambda}$
I_{xy}	$\frac{V_\Delta}{D_1}$	0	0

Notations

$$D_1 = T_\Delta V_\lambda - V_\Delta^2$$
$$D_2 = T_\theta V_\Delta - V_\theta T_\Delta$$
$$D_3 = T_\theta V_\lambda - V_\theta V_\Delta$$
$$D_4 = M_\theta T_\Delta - T_\theta^2$$
$$D_5 = M_\theta V_\lambda - V_\theta^2$$
$$D = M_\theta T_\Delta V_\lambda - M_\theta V_\Delta^2 - T_\Delta V_\theta^2 - V_\lambda T_\theta^2 + 2 V_\theta T_\theta V_\Delta$$

TABLE (2) Total Joint Displacements.

	Joint B			Joint F	
	θ	Δ		θ	Δ
Balancing Displacement at B and induced displacement at F	5.32	-	→	1.36	9.00
	-	90.9	→	2.14	12.90
Balancing displacement at F and induced displacement at B	0.60	3.94	←	2.34	-
	0.38	2.26	←	-	13.90
Total Displacements	6.30	97.1		5.84	37.8

TABLE (3). Moment Computations.

	JOINT B				JOINT F		
	BK	BJ	BF		FB	FI	FH
Fixed End Moment	2.6	0.0	-.581		-.326	0	0
Rotation $\theta^B = 6.3$	239.4	504	382		-.273	-	-
Rotation $\theta^F = 5.84$	-	-	-.253		354	469	221
Trans. $\Delta^B = 97.1$	194	-.728	388		-.388	-	-
Trans $\Delta^F = 37.8$			-.151		151	-.284	76
Final Moments Σ	436	-.224	-.215		-.482	185	297

TABLE (4) Computations of Thrusts T

	JOINT B				JOINT F		
	BK	BJ	BF		FB	FI	FH
Fixed End Thrust	-.89.3	0	-.118.1		-.31.9	0	0
Rotation $\theta^B = 6.3$	12.6	-.47.3	25.2		-.25.2	-	-
Rotation $\theta^F = 5.84$	-	-	-.23.4		23.4	-.43.8	11.7
Trans. $\Delta^B = 97.1$	129.5	91.2	32.3		-.32.3	-	-
Trans. $\Delta^F = 37.8$	-	-	-.12.6		12.6	35.4	50.4
Final Thrusts	52.8	43.9	-.96.5		-.53.4	-.8.4	62.1

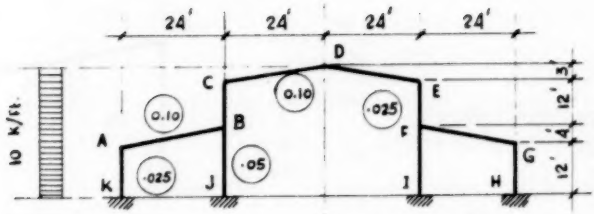


FIG. 1 - $\bar{S}$ values thus (1)

Elastic Constants:

$$\bar{S} = \int \frac{ds}{EI} = \int d\bar{s}$$

$$I_x = \int y^2 \, d\bar{s}$$

$$I_y = \int x^2 \, d\bar{s}$$

$$I_{xy} = \int xy \, d\bar{s}$$

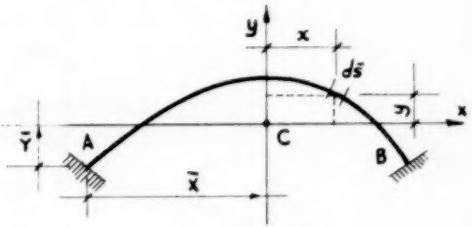


FIG. (2)

Elastic Constants.

$\bar{S}$ = 0.250

I_x = 3.00 = I'_x

I_y = 67.20 = I'_y

I_{xy} = 0.0

Stiffness Factors

(End B)

T_Δ = 0.333

T_θ = 4.00 = M_Δ

V_θ = -0.357

M_θ = 60.57

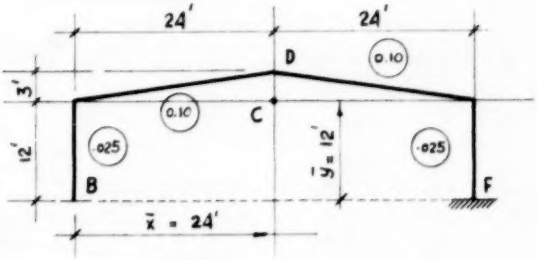
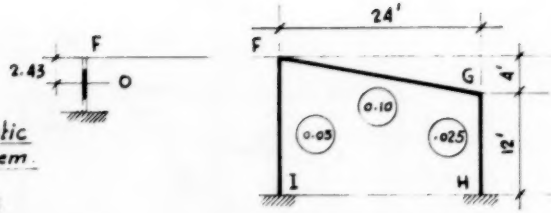


FIG. (3)

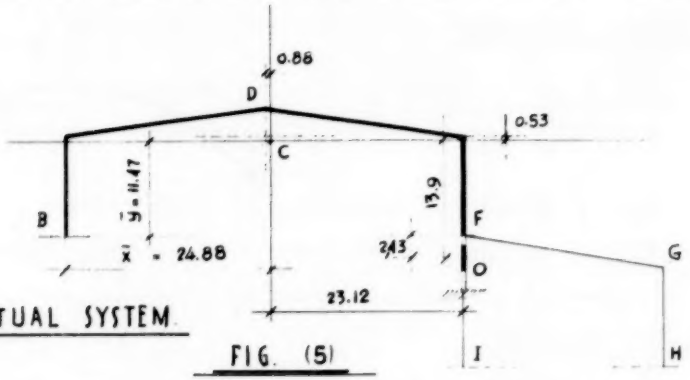


a. Real System

b. Equivalent Elastic Symbolic System.

$\bar{S} = 0.0095$
$I_x = 0.44$
$\bar{y} = -2.43$
$I_y = I_{xy} = \bar{x} = 0$

FIG. (4)



VIRTUAL SYSTEM.

FIG. (5)

A. Elastic Constants.

$\bar{S} = 0.2595$
$I_x = 5.35$
$I_y = 72.5$
$I_{xy} = -3.18$

then :

$I'_x = 5.20$
$I'_y = 70.50$

B. Stiffness Factors (End B)

$T_\Delta = 0.192$
$V_\Delta = -0.0085$
$M_\Delta = 2.41 = T_\theta$
$V_\theta = -0.45$
$M_\theta = 42.80$

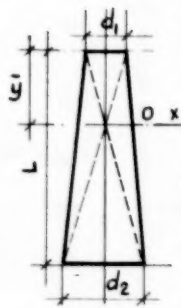
C. Induced Displacement (at F)

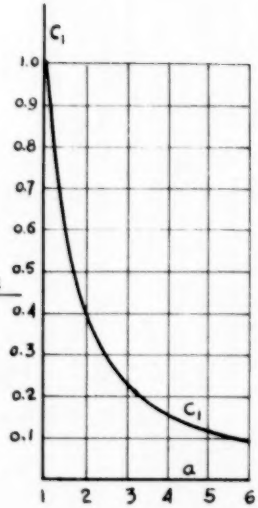
$\Delta_\Delta = 0.142$ $\theta_\Delta = 0.0235$
$\Delta_\theta = 1.68$ $\theta_\theta = 0.257$

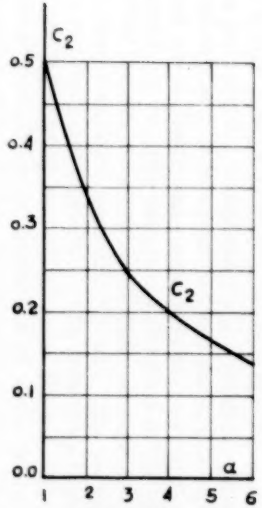
$\bar{S} = C_1 \cdot \frac{L}{EI_1}$

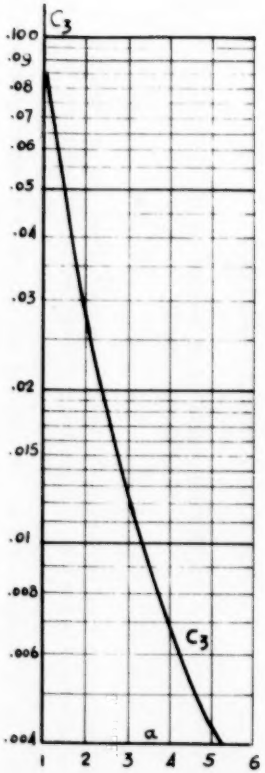
$\bar{y} = C_2 \cdot L$

$I_x = C_3 \frac{L^3}{EI_1}$









$a = d_2/d_1 = \sqrt[3]{I_2/I_1}$

$C_1 = (1+a)/2a^2$

$C_2 = 1/(1+a)$

$C_3 = \frac{1}{(a-1)^3}\left[\log a - \frac{2(a-1)}{(a+1)}\right]$

TAPERED BEAMS

ELASTIC CONSTANTS

FIGURE (6)

Discussion of
"CONTINUOUS PRESTRESSING"

by Robert B. B. Moorman ASCE
(Proc. Paper 588)

A. DIWAN, A. M. ASCE.[1]—The concept of equivalent load representing the effects of prestressing in the case of continuous prestressed beams is seen to be of a rather limited value. Indeed, it is only useful in the case when the prestressing cable is such that the function of the eccentricity "e" is a second degree parabola, in which case the equivalent load will have a constant value.

Even in this case, it is also necessary that the coefficients for fixed end moments for partially loaded beams be available. However, it cannot be stated that this will be the most suitable case, although it may be the most convenient one to justify the application of the equivalent load concept. Mr. Magnel[2] for example suggests a parabola of the fourth degree for the shape of the prestressing cable. In this, and similar cases, the equivalent load will have a variable intensity, for which the coefficients for the fixed end moments are not readily available. To determine these moments we have to re-consider the bending moment diagram for any chosen statically determinate main-system, the most convenient of which is a system of simple spans. For any simple span however, the bending moment diagram due to prestressing is directly given by the curve enclosed by the cable and the axis of the beam, multiplied by the resultant (horizontal) tension in the cable. With this curve representing the bending moment diagram for the statically determinate system, the final B.M.D. for the continuous beam can be readily computed by any of the classical methods. Of these however, the most suitable seems to be that using the Clapyron's equation of three moments.

Consider now the two examples presented by the author, Fig. (1-a) shows the first of these examples, with the prestressing cable taking the shape of a single second degree parabola. The total elastic load on each span will be;

$$EI.\ P = -\frac{2}{3} L e T$$

The elastic reaction at support B with both spans loaded will be:

$$R^B = \frac{5}{4} P$$

$$= -\frac{5}{6}\left(\frac{T e L}{EI}\right)$$

1. Structural Dept., Faculty of Eng., Alexandria Univ., Alexandria, Egypt.
2. "Prestressed Concrete"—Concrete Publications. P. 84.

From the equation of three moments:

$$4\,M\bar{L} = -6\,R^B$$

or

$$M = 1.25\,Te$$

The net moment at B will be:

$$M^B = Te\,(1.25 - 1.0)$$
$$= 0.25\,Te$$

This equals:

$$M^B = 0.25 \times 610 \times 24.5 / 12$$
$$= 311 \quad k.\,ft.$$

The B.M.D. is shown in figure (1-b).

For the second case of the prestressing cable, the elastic loads will be as shown in figure (1-c).

The elastic reaction at the middle support B will be:

$$EI\,R^B = -2.8\,T \qquad k.\,ft^2$$

The continuity moment necessary at B will be:

$$M = \frac{6(2.8)}{4(70)}\,T$$
$$= 0.06\,T$$

The resultant moment at B will then be:

$$M^B = (1.727 + .060)T$$
$$= 1.787\,T$$

For $T = 610$ kips.

$$M = 1090.5 \quad k.\,ft.$$

Now let us consider a more complicated shape for the curve of the prestressing cable, as shown in figure (2) which represents a continuous beam of three equal spans. From A to B the prestressing cable follows a second degree parabola, while the shape of the curve between B.C. and C-D is given by a fourth degree parabola. Figure (2) shows two types for the prestressing cable. In type (2), all eccentricities "e" are positive while in type (1), the eccentricity e_2 at the middle supports is negative:

Considering part BC; the equation of the parabolic curve will be,

$$e = e_1 + \frac{8x^2}{L^4}(e_1 - e_2)(2x^2 - L^2) \qquad \ldots (1)$$

In which (X) is measured from the mid-span B towards the support C; and (e) is positive below the axis of the beam.

The area of the curve will be:

$$A = \int_0^{\frac{L}{2}} e \, dx$$

$$= \frac{L}{30} (8e_1 + 7e_2) \quad \ldots\ldots (2)$$

The centroid of the enclosed area will lie to the right of B at a distance $(\bar{X})$, such that

$$\bar{X} = \frac{\int_0^{\frac{L}{2}} e x \, dx}{A} = \frac{5(e_1 + 2e_2)}{4(8e_1 + 7e_2)} \quad \ldots (3)$$

The total elastic reaction at C due to the elastic loading on both spans will be:

$$EI \; R^C = - \frac{LT}{240} (67 e_1 + 104 e_2 + 64 e_3) \ldots (4)$$

Thus the continuity moment there will be:

$$5\bar{L} \; M^C = - 6 R^C$$

From which:

$$M^C = T (.335 e_1 + 0.52 e_2 + 0.32 e_3) \ldots\ldots (5)$$

The equations apply to both types of prestressing cable provided the appropriate sigh is taken for "e."

It is also interesting to study the effect of the external loading on the tension in the prestressing cable, although it is true that for most practical cases this effect is neglibible.

A prestressed one-span beam simply supported is in fact once internally statically indeterminate. A main system for such a beam may be obtained by cutting the prestressing cable somewhere. Now consider the case of a straight prestressing cable. Using the method of "virtual work," the force produced in the cable due to a uniformly distributed load covering the full span L of the beam will be:

$$T = \frac{2e}{3C} \cdot \frac{pL^2}{8}$$

where: e = constant eccentricity of cable.

$$C = \left(e^2 + \frac{I}{n A_s} + \frac{I}{A} \right)$$

and:

A,I are the area and moment of inertia respectively for the concrete section.

A_s = area of prestressing cable.

$n = E_s \div E_c$

Using the same cross section of the author, and assuming:

A	= 627	in^2
I	= 254.750	in^4
e	= 25	in
A_s	= 6	in^2
n	= 7	

then:

C = 7080 in^2

The moment produced by the developing tension will be:

$$M = -\frac{2e^2}{3C} \left(\frac{pL^2}{8} \right)$$

$$= -0.06 \left(\frac{pL^2}{8} \right)$$

Thus the maximum moment is reduced by about 6% due to the additional force produced in the prestressing cable.

The case of a two-span continuous beam with a straight prestressing cable is also shown in figure (3). Here the problem is twice statically indeterminate.

Due to a uniformly distributed load, covering the two spans, and taking as unknowns the force T developing in the cable and the moment M_1 necessary to re-establish the continuity at the middle support we get:

$$T = \frac{2e}{3(4C - 3e^2)} \left(\frac{pL^2}{8} \right)$$

$$M_1 = \frac{4(C - e^2)}{(4C - 3e^2)} \cdot \frac{pL^2}{8} \qquad \text{negative.}$$

Using the same dimensions given before we have:

$$-T.e = -0.016 \left(\frac{pL^2}{8} \right)$$

$$+M_1 = -0.975 \left(\frac{pL^2}{8} \right)$$

The total moment at the middle support will therefore be:

$$M^B = -(.975 + .016) \frac{pL^2}{8} = -.991 \frac{pL^2}{8}$$

Here the total effect contributed by the additional tension in the cable is much less than that previously obtained for the single span beam. This is obviously due to the fact that the prestressing cable runs in regions of both compressive and tensile stresses in the continuous beam, which greatly reduces the final elongation necessary in the cable. However, this will not be the case for the cable used in figures (2-1) and (1-c) where the full length of the cable almost lies in the region of tensile stresses. In these cases however, the integrations necessary in the solution may be obtained by a summation process.

L.J. MENSCH,[3] M. ASCE.—The novel art of prestressed concrete has produced a flood of papers on the strength of prestressed beams and slabs, based mostly on the laws of elasticity of isotropic materials following Hooke's law. But the material concrete is neither isotropic nor does it follow Hooke's law under stress and the careful designer of prestressed concrete should hesitate using the rules and formulas obtained by the theory of elasticity and should try to find the factors of correction which are to be used to modify the theoretical formulas, from tests made by experienced investigators, unbiased by the fame of the mathematical theory of elasticity.

Such tests are expensive and take time.

A number of these factors have been known for a long time. In the case of a prismatic plain concrete beam of rectangular section the theoretical bending moment at the ultimate load is given by the formula $M_{ult} = f_t bd^2/6$ where f_t is the unit strength in tension. But many careful tests have shown that the ultimate bending moment is much greater than given by above formula and it has been customary to assume that there is a good agreement with tests if we use a fictive tensile strength of twice the value of the actual tensile strength of the concrete in the theoretical formula. What the compressive stresses are at the ultimate load or at a fraction of this load has not yet been found. In the writer's opinion there cannot exist a higher tensile strength than that obtained by careful tensile tests and that the assumed value of resisting moment $bd^2/6$ is at fault. The resisting moment for tension becomes larger if the neutral axis approaches the compression face and this will make the resisting moment for compression smaller and therefore the maximum compression will be much larger.

Another important case for which the factor of correction has been found is the case of excentrically loaded columns from tests by Prof. Considere and by Prof. Bach on plain and reinforced columns, 50 and 40 years ago, respectively, which tests have shown that the columns fail at a load from 30% to 50% greater than derived from theory.

For the continuous beam on three supports with two equal spans which is the subject of this paper, tests by Prof. E. Probst & Prof. Scheit, about 40 years ago, have shown that the ultimate moment at the center support are given by the moment $wL^2/10$ instead of the theoretical moment $wl^2/8$, used in the paper.

The engineer who uses the author's method of designing a prestressed structure will produce a safe design but will not be able to guess its actual strength by a wide margin.

3. Civ. Engr. and Contr., Evanston, Ill.

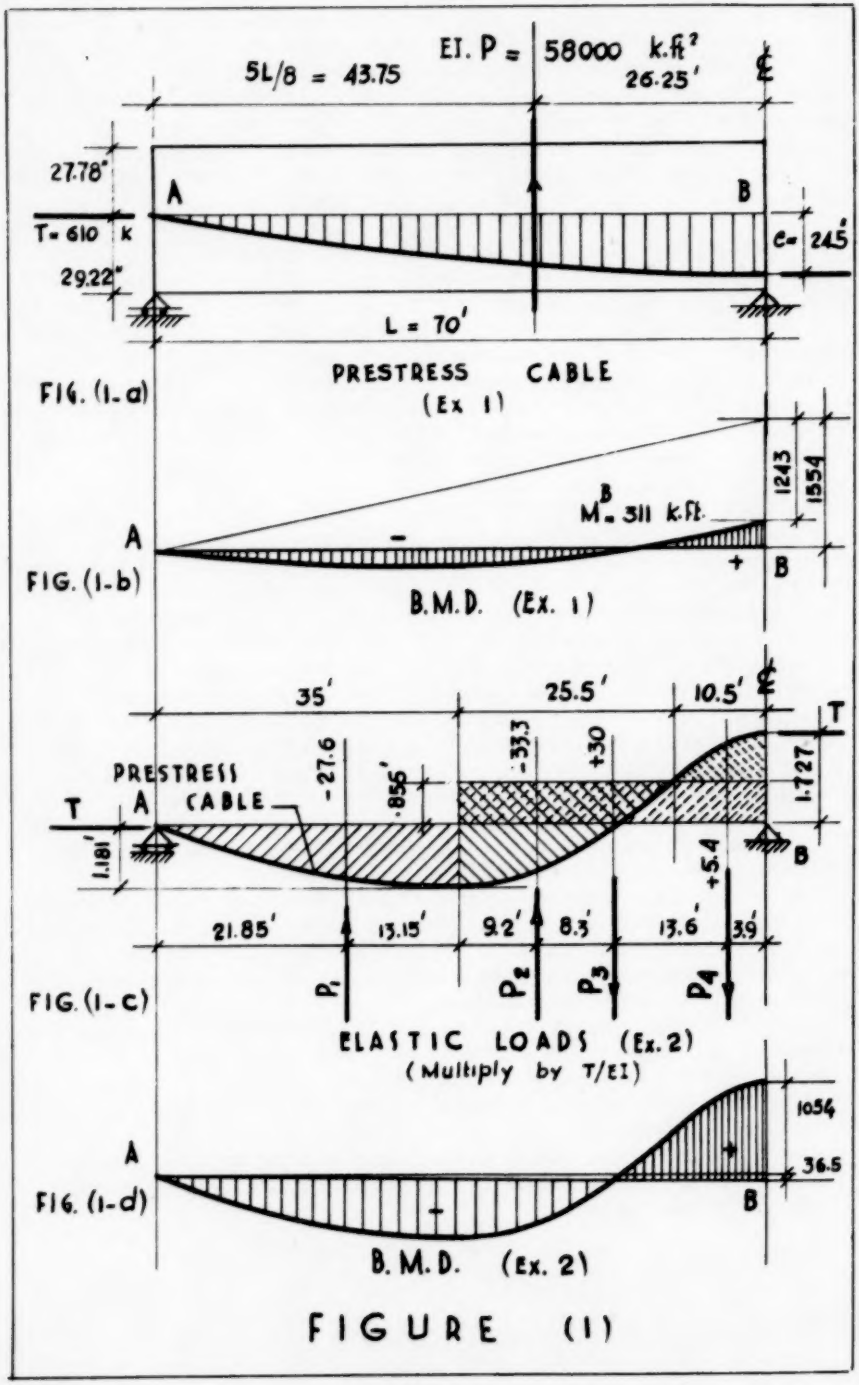

EI. P = 58000 k.ft²
5L/8 = 43.75
26.25'
27.78"
A
B
T = 610 k
29.22"
e = 24.5"
L = 70'
PRESTRESS CABLE
(Ex 1)
FIG. (1-a)
1243
1554
M_B = 311 k.ft.
A
B
FIG. (1-b)
B.M.D. (Ex. 1)
35'
25.5'
10.5'
T
PRESTRESS CABLE
-27.6
-333
+30
.856'
1.727
T
A
B
1.181'
+5.4
21.85'
13.15'
9.2'
8.3'
13.6'
3.9'
P1
P2
P3
P4
FIG. (1-c)
ELASTIC LOADS (Ex. 2)
(Multiply by T/EI)
1054
36.5
A
B
FIG. (1-d)
B.M.D. (Ex. 2)

FIGURE (1)

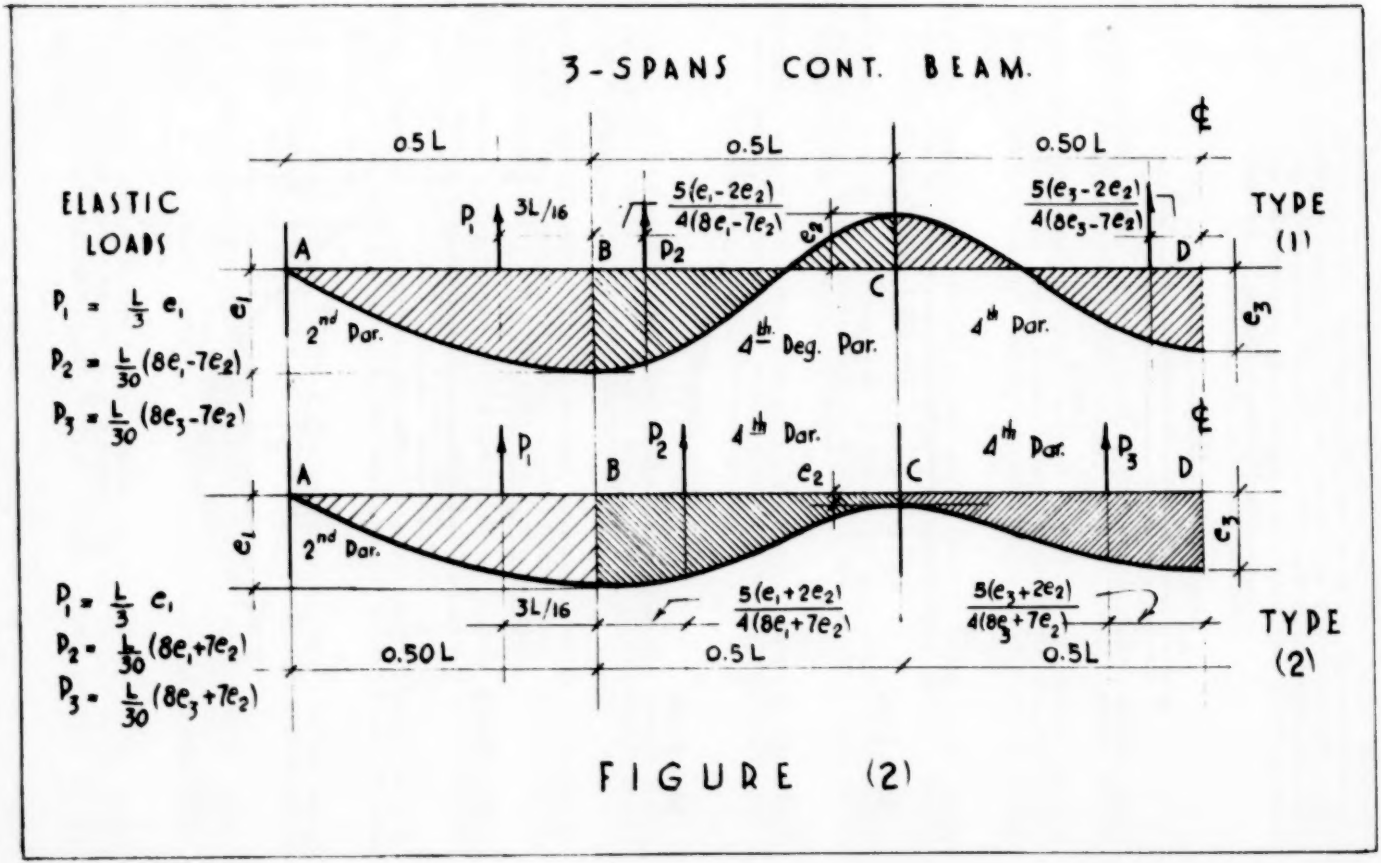

3-SPANS CONT. BEAM.
0.5L
0.5L
0.50L
ELASTIC LOADS
$P_1 = \frac{L}{3} e_1$
$P_2 = \frac{L}{30}(8e_1 - 7e_2)$
$P_3 = \frac{L}{30}(8e_3 - 7e_2)$
P_1
3L/16
$\frac{5(e_1 - 2e_2)}{4(8e_1 - 7e_2)}$
P_2
$\frac{5(e_3 - 2e_2)}{4(8e_3 - 7e_2)}$
TYPE (1)
A
B
C
D
e_1
e_2
e_3
2nd Par.
4th Deg. Par.
4th Par.
4th Par.
4th Par.
P_1
P_2
P_3
$P_1 = \frac{L}{3} e_1$
$P_2 = \frac{L}{30}(8e_1 + 7e_2)$
$P_3 = \frac{L}{30}(8e_3 + 7e_2)$
3L/16
$\frac{5(e_1 + 2e_2)}{4(8e_1 + 7e_2)}$
$\frac{5(e_3 + 2e_2)}{4(8e_3 + 7e_2)}$
TYPE (2)
0.50L
0.5L
0.5L

FIGURE (2)

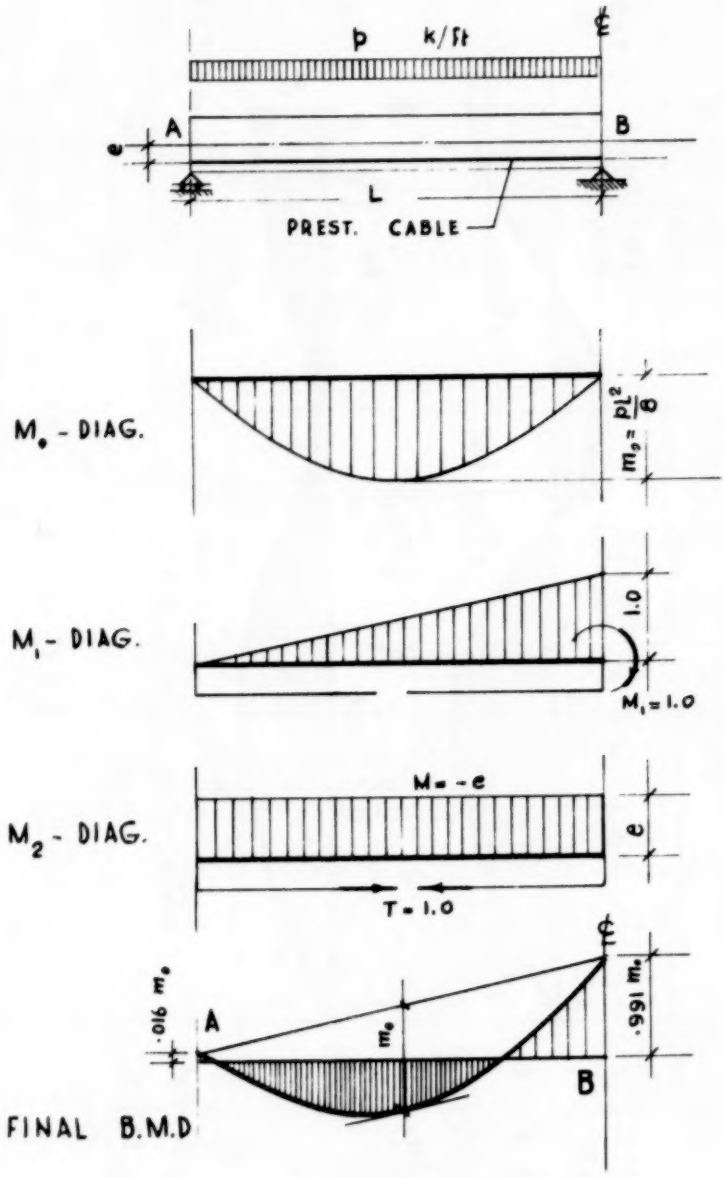

p k/ft
A
B
e
L
PREST. CABLE
M₀ - DIAG.
m₀ = pl²/8
M₁ - DIAG.
1.0
M₁ = 1.0
M₂ - DIAG.
M = -e
e
T = 1.0
FINAL B.M.D
·916 m₀
m₀
·166 m₀
A
B

FIGURE (3)

PROCEEDINGS PAPERS

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 80 (1954)

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

VOLUME 81 (1955)

JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)[c], 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)[c], 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)[c], 700(PO), 701(ST)[c].

JUNE: 702(HW), 703(HW), 704(HW)[c], 705(IR), 706(IR), 707(IR), 708(IR), 709(HY)[c], 710(CP), 711(CP), 712(CP), 713(CP)[c], 714(HY), 715(HY), 716(HY), 717(HY), 718(SM)[c], 719(HY)[c], 720(AT), 721(AT), 722(SU), 723(WW), 724(WW), 725(WW), 726(WW)[c], 727(WW), 728(IR), 729(IR), 730(SU)[c], 731(SU).

JULY: 732(ST), 733(ST), 734(ST), 735(ST), 736(ST), 737(PO), 738(PO), 739(PO), 740(PO), 741(PO), 742(PO), 743(HY), 744(HY), 745(HY), 746(HY), 747(HY), 748(HY)[c], 749(SA), 750(SA), 751(SA), 752(SA)[c], 753(SM), 754(SM), 755(SM), 756(SM), 757(SM), 758(CO)[c], 759(SM)[c], 760(WW)[c].

AUGUST: 761(BD), 762(ST), 763(ST), 764(ST), 765(ST)[c], 766(CP), 767(CP), 768(CP), 769(CP), 770(CP), 771(EM), 772(EM), 773(SA), 774(EM), 775(EM), 776(EM)[c], 777(AT), 778(AT), 779(SA), 780(SA), 781(SA), 782(SA)[c], 783(HW), 784(HW), 785(CP), 786(ST).

c. Discussion of several papers, grouped by Divisions.

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